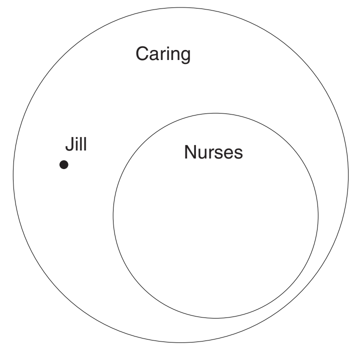
MA 110-91 §1.1 – 3.1	**Test #1**	*score*	**Name:** ____________ **30 September 2000**

1. Use a properly labeled Venn diagram to determine the validity of the following argument. Explain. *(9 points)*

   1. All nurses are caring.
   2. Jill is caring.

   Therefore Jill is a nurse.



Solution: From the Venn diagram, we see Jill can be caring while not being a nurse.

2. Convert the following argument into symbolic form and determine if the argument is valid using a truth table. *(10 points)*

   If math is interesting, then I'll get an A. If I study well, then I'll get an A. I get an A. Therefore, I studied well or math is interesting.

Solution:

p	q	r	$p \to q$	$r \to q$	$(p \to q) \wedge (r \to q) \wedge q$	$r \vee p$	$(p \to q) \wedge (r \to q) \wedge q \to (r \vee p)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	F	F
F	F	T	T	F	F	T	T
F	F	F	T	T	F	F	T

The last column does not have all T's, so the argument is invalid.

3. Write the contrapositive of the statement "If I get an A or a B then I will be happy" *(8 points)*

Solution: "If I am not happy, then I didn't get an A or a B."

4. If $U = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$, $A = \{1,3,5,7,9\}$ and $B = \{1,2,3,4,5,6\}$, find the set $A \cap B$; then find $(A' \cup B')'$. *(9 points)*

Solution: Checking the common elements between A and B, we see that $A \cap B = \{1,3,5\}$. For the second part, we can use DeMorgan's Laws to get $(A' \cup B')' = (A')' \cap (B')' = A \cap B = \{1,3,5\}$.

5. How many subsets are there of a set having 8 elements? Explain how the answer is related to the number of ways a hamburger can be served if there are 8 different condiments for the customer to choose from. *(9 points)*

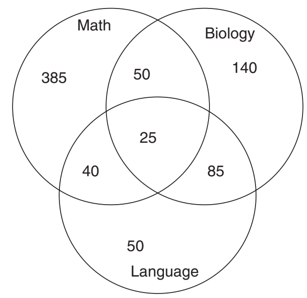
Solution: Perform 8 successive events where on the first event you choose whether or not to select the first elemet on the set, on the second event you choose whether to select the second element, ... Each of the 8 events has two outcomes, so the total is 2^8. Thus, if you have 8 condiments available for a hamburger, you have $2^8 = 256$ different hamburgers you can assemble since a selection of condiments corresponds to choosing a subset of the 8 condiments.

6. In a dorm holding 1000 students, 500 are taking math, 300 are taking biology, and 200 are taking a foreign language. Suppose 25 are taking all three, 75 are taking math and biology, 50 are taking only a foreign language, and 65 are taking math and a foreign language. What percentage of students in the dorm are taking at least one of these three subjects?

   Draw and label a Venn diagram and explain your reasoning. *(10 points)*

   Solution:

   By filling in the information in the Venn diagram (below), we see that there are 225 students who aren't in any of the three sets. So there are $1000 - 225 = 775$ students taking at least one of the courses.



7. Compute the numbers ${}_8P_5$ and ${}_8C_5$. Explain how these numbers would be used in a counting argument. *(9 points)*

   Solution:
   ${}_8P_5 = \dfrac{8!}{(8-5)!} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720$ and ${}_8C_5 = \dfrac{8!}{(8-5)! \cdot 5!} = \dfrac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{5!} = 56$. The permutation ${}_8P_5$ would be used in a counting argument where you wanted to choose 5 objects from 8 objects where the order mattered. The combination ${}_8C_5$ would be used in a counting argument where you wanted to choose 5 objects from 8 objects where the order did not matter.

8. In how many ways can teams in a 10-team league finish first, second, and third (assuming no ties). Explain your counting argument. *(9 points)*

   Solution: We need to select three teams from the ten and the order is important, so the answer is ${}_{10}P_3 = 10 \cdot 9 \cdot 8 = 720$.

9. From a group of 7 men and 8 women, how many different ways are there to choose a 5-person committee consisting of 3 women and 2 men? Explain your counting argument. *(9 points)*

   Solution: We can choose the women in ${}_8C_3 = 56$ ways and the men in ${}_7C_2 = 21$ ways. We need to do both events in succession, so the total number of ways will be the product $56 \cdot 21 = 1287$.

10. How many different 5-card poker hands are a flush (i.e., all cards in the same suit)? *(9 points)*

    Solution: We do two events: choose the suit (4 ways) and then choose 5 cards from the 13 cards in the choosen suit (${}_{13}C_5) = 1287$), so the total is $4 \cdot 1287 = 5148$.

11. What is the probability of rolling a total of 8 on two rolls of a die? Explain you reasoning. *(9 points)*

    Solution: Make a 6×6 rectangular array with the ordered pairs representing all possible rolls. We find that 5 of the ordered pairs add to 8. So the probability is $\frac{5}{36} \approx 14\%$.