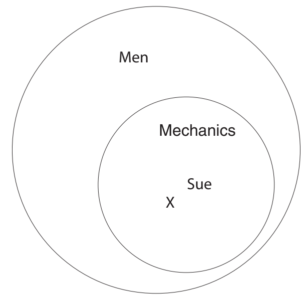
MA 110-06 §1.1 – 3.2	# Test #1 Solutions	**Name:** ____________ 4 October 2001

1. Use a properly labeled Venn diagram to determine the validity of the following argument. Explain. *(9 points)*

    1. All mechanics are men.
    2. Sue is a mechanic.

    Therefore, Sue is a man.

    Solution:

    From the Venn diagram, we see that since Sue is a mechanic and all mechanics are men, Sue must indeed be a mechanic. The argument is VALID.

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2. Construct a truth table to show that the inverse of the symbolic statement $p \to q$ is logically equivalent to its converse. *(9 points)*

    Solution: The converse of the statement is $q \to p$ and the inverse is $\sim p \to \sim q$.

p	q	$\sim p$	$\sim q$	$q \to p$	$\sim p \to \sim q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	T	T

Since the truth tables are the same, the two statements are logically equivalent.

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3. Write the following argument in symbolic form. Then use a complete truth table to determine if the argument is valid. *(9 points)*

    If the defendant goes to jail, then the defendant is not innocent. If the defendant's lawyer is good, then the defendant does not go to jail. The defendant does not go to jail. Therefore, the defendant is innocent or the lawyer is good.

    Solution: Use the symbolic representations p: the defendant goes to jail; q: the defendant is innocent; r: the lawyer is good. Then the paragraph can be written as $\big((p \to \sim q) \wedge (r \to \sim p) \wedge \sim p\big) \to (q \vee r)$ which turns out to have one F in the truth table. So the argument is invalid.

p	q	r	$\sim p$	$\sim q$	$p \to \sim q$	$r \to \sim p$	$(p \to \sim q) \wedge (r \to \sim p) \wedge \sim p$	$q \vee r$	$\big((p \to \sim q) \wedge (r \to \sim p) \wedge \sim p\big) \to (q \vee r)$
T	T	T	F	F	F	F	F	T	T
T	T	F	F	F	F	T	F	T	T
T	F	T	F	T	T	F	F	T	T
T	F	F	F	T	T	T	F	F	T
F	T	T	T	F	T	T	T	T	T
F	T	F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	F	F

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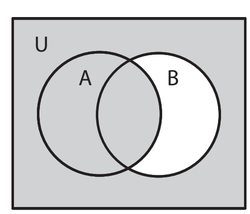
4. If $U = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n\}$, $A = \{a, b, c, g, k, l\}$ and $B = \{a, c, e, f, g, k, m\}$, find the set $A \cup B'$. Then illustrate $A \cup B'$ by shading the result in a Venn diagram. *(9 points)*

    Solution: The set $A \cup B'$ consists of all the elements in A together with those that are not in B. So, $A \cup B' = \{a, b, c, d, g, h, i, j, k, l, n\}$.

    The Venn diagram for $A \cup B'$ is shown to the right.

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5. List all the subsets of the set $\{a, b, c\}$. In general, if A is a set with $n(A) = k$, how many subsets of A are there? Explain. *(9 points)*

Solution: There are 8 subsets of the given set: $\{\}$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, and $\{a, b, c\}$. In general, a set with n elements has 2^n subsets. That is because the subsets of the given set can all be constructed as follows: for each of the n elements in the given set, decide whether or not it is admitted into the subset. That gives a total of n events, each with 2 outcomes – 2^n.

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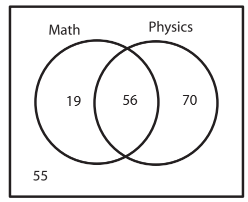
6. In a group of 200 students, 75 enjoy reading math books, 126 enjoy reading physics books, and 55 don't enjoy books of either subject (although they wish they did). How many of the students enjoy reading both math and physics books?

   Draw a properly labeled Venn diagram and explain your reasoning. *(9 points)*

Solution:

Since 55 of the 200 students play neither, that means $n(M \cup P) = 200 - 55 = 145$. But we also know $n(M \cup P) = n(M) + n(P) - n(M \cap P)$, so $145 = 75 + 126 - n(T \cap R)$. Solving for the intersection, we see 56 students enjoy both.

The Venn diagram is shown to the right.



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7. From a group of 10 people, in how many ways can 4 of them be selected and lined up in a row? In how many ways can 4 of the 10 be selected for membership on a committee? *(9 points)*

Solution: The number of ways to select 4 and line them up is ${}_{10}P_4 = 10 \cdot 9 \cdot 8 \cdot 7 = 5,040$

The number of ways to select 4 without order is ${}_{10}C_4 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210$.

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8. You are arranging your opera compact disks on a shelf at home. There are 3 CDs of Puccini, 4 of Wagner, and 5 of Verdi. In how many ways can they be arranged on the shelf? In how many ways can they be arranged if CDs of the same composer must be adjacent? Explain. *(10 points)*

Solution: Twelve CDs can be arranged on a shelf in $12! = 479,001,600$ ways.

To keep the same composers adjacent, we first pick an order for the three composers (3! ways), then arrange the Puccini CDs (3!), then the Wagner (4!), and finally the Verdi (5!). Taking the product, we get $3! \cdot 3! \cdot 4! \cdot 5! = 103,680$.

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9. From a group of 7 men and 10 women, how many different ways are there to choose a 5-person committee consisting of 3 women and 2 men? Explain your counting argument. *(9 points)*

Solution: First we can choose 2 men from 7 (${}_7C_2$) and then choose 3 women from 10 (${}_{10}C_3$). Taking the product, we get ${}_7C_2 \cdot {}_{10}C_3 = 2,520$.

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10. From a standard 52-card deck, how many different 5-card hands consist of four cards in one suit with the fifth card in a different suit? Explain. *(9 points)*

Solution: First we select a suit (4 ways), then four cards from the selected suit (${}_{13}C_4$), and finally one remaining card (39). Taking the product, we get $4 \cdot {}_{13}C_4 \cdot 39 = 111,540$.

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11. A coin is flipped three times and the result of each flip is recorded. Write out the sample space for this three-flip experiment. Then find the probability that the three flips result in more heads than tails. Explain. *(9 points)*

Solution: Sample space: $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$. Since four of these eight outcomes have more heads than tail,

$$p(\text{more heads than tails}) = \frac{4}{8} = 0.5$$