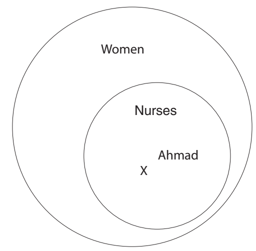
MA 110-06 **§1.1 – 3.2**	**Test #1**	*score*	**Name:** ____________________ **3 October 2002**

1. Use a properly labeled Venn diagram to determine the validity of the following argument. Explain. *(9 points)*

   1. All nurses are women.
   2. Ahmad is a nurse.

   Therefore, Ahmad is a woman.



Solution:

From the Venn diagram, we see that since Ahmad is a nurse and all nurses are women, Ahmad must indeed be a woman. The argument is VALID.

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2. Construct a truth table to show that the symbolic statement $p \to q$ is logically equivalent to its contrapositive. *(9 points)*

Solution:

The contrapositive of the statement $p \to q$ is $\sim q \to \sim p$.
Since the truth tables are the same, the two statements are logically equivalent.

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

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3. Write the following argument in symbolic form. Then use a complete truth table to determine if the argument is valid. *(9 points)*

   All hurricanes produce dangerous weather. Whenever there is dangerous weather, classes are canceled. Classes are not canceled. Therefore, there is no hurricane.

Solution: Use the symbolic representations p: there is a hurricane; q: there is dangerous weather; r: classes are canceled. Then the paragraph can be written as $\big((p \to q) \wedge (q \to r) \wedge \sim r\big) \to \big(\sim p\big)$ which turns out to have all T's in the truth table. So the statement is a tautology and the argument is valid.

p	q	r	$\sim p$	$\sim r$	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r) \wedge \sim r$	$\big((p \to q) \wedge (q \to r) \wedge \sim r\big) \to \sim p$
T	T	T	F	F	T	T	F	T
T	T	F	F	T	T	F	F	T
T	F	T	F	T	F	T	F	T
T	F	F	F	T	F	T	F	T
F	T	T	T	F	T	T	F	T
F	T	F	T	F	T	F	F	T
F	F	T	T	T	T	T	F	T
F	F	F	T	T	T	T	T	T

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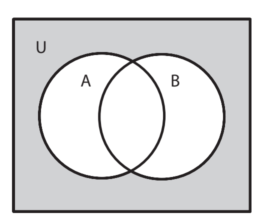
4. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$, $A = \{1, 3, 4, 6, 8, 11\}$ and $B = \{2, 4, 6, 8, 12, 14\}$, find the set $(A \cup B)'$. Then illustrate the set $(A \cup B)'$ by shading the result in a Venn diagram. *(9 points)*

Solution:

$A \cup B = \{1, 2, 3, 4, 6, 8, 11, 12, 13\}$, so $(A \cup B)' = \{5, 7, 9, 10, 13, 15\}$



5. Let $A = \{a, b, c, d\}$. List all the subsets of A that have cardinality two. How can you tell how many such subsets there are (using a combination or permutation number) without listing them? Explain. *(9 points)*

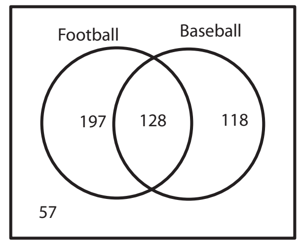
   Solution: The subset of cardinality 2 are: $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$. There are ${}_4C_2 = 6$ such subsets.

6. In a group of 500 people, 325 enjoy watching football, 246 enjoy watching baseball, and 57 don't enjoy watching either sport. How many of the people enjoy watching both sports?

   Draw a properly labeled Venn diagram and explain your reasoning. *(9 points)*

   Solution:

   Let F = the set of people who enjoy watching football and B = the set of people who enjoy watching baseball. Then $|F \cup B| = |F| + |B| - |F \cap B|$. So $500 - 57 = 325 + 246 - |F \cap B|$. Solving for $|F \cap B|$ yields 128.



7. From a group of 10 people, in how many ways can they be lined up in a row? In how many ways can they be lined up if a given pair of people must be adjacent? *(9 points)*

   Solution: Lining up all 10 people in a row gives ${}_{10}P_{10} = 10! = 3,628,800$ ways.

   Keeping a given pair adjacent gives $2! \cdot 9! = 725,760$ ways.

8. How many different license tags can be generated if they consist of three capital letters followed by three digits (zero through nine) if none of the letters can be repeated? Explain. *(10 points)*

   Solution: ${}_{26}P_3 = 26 \cdot 25 \cdot 24 \cdot 10 \cdot 10 \cdot 10 = 15,600,000$

9. From a group of eight men and nine women, in how many ways can a committee of five be chosen? In how many ways can such a committee be chosen if there must be at least two of each gender? *(9 points)*

   Solution: To choose a committee of 5, we select 5 people from 17 without regard to order: ${}_{17}C_5 = 6,188$

   To form committees that have at least 2 of each gender, we count the 2 men and 3 women committees and add the number of 3 men and 2 women committees: ${}_8C_2 \cdot_9 C_3 +_8 C_3 \cdot_9 C_2 = 28 \cdot 84 + 56 \cdot 36 = 2,352 + 2,016 = 4,368$

10. From a standard 52-card deck, how many different 5-card hands consist of three cards in one suit and two cards in a different suit? Explain. *(9 points)*

    Solution: Structure the count as a sequence of 4 events:

    (a) select a suit (4 ways);
    (b) select 3 cards from the selected suit (${}_{13}C_3$ ways);
    (c) selct a different suit (3 ways);
    (d) select 2 cards from the selected suit (${}_{13}C_2$ ways).

    Multiply the number of ways each event can be performed together to get 267,696

11. A coin is flipped four times and the result of each flip is recorded. Write out the sample space for this four-flip experiment. Then find the probability that the four flips result in more heads than tails. Explain. *(9 points)*

Solution: There are 16 outcomes in the sample space:

HHHH
HHHT
HHTH
HHTT
HTHH
HTHT
HTTH
HTTT
THHH
THHT
THTH
THTT
TTHH
TTHT
TTTH
TTTT

There are 5 of these 16 outcomes that have more heads than tails, so $p(\text{more heads than tails}) = \frac{5}{16} \approx .31$.