MA 110-06 §3.3 – 4.4	**Test #2**	*score*	**Name:** ________________ **21 November 2002**

1. If two dice are rolled, find the probability that the sum of the dice is 8. *(9 points)*

   Solution:

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

   This was supposed to be an easy warmup problem. From the table, you count five ways the dice can sum to 8, so $p(\text{sum of the dice is 8}) = \frac{5}{36} \approx 0.14$

2. Find the probability of being dealt four cards in the same kind in a five-card hand. Express your answer as a decimal number. *(9 points)*

   Solution: We count the number of four-of-a-kind hands by choosing a kind (13 ways), choosing 4 cards from the chosen kind (${}_4C_4 = 1$ was), and choosing a fifth card from the remaining 48 (${}_{48}C_1 = 48$ ways). So $p(\text{4-of-a-kind}) = \frac{13\cdot 1\cdot 48}{{}_{52}C_5} = \frac{624}{2{,}598{,}960} \approx 0.0002$.

3. We play a lottery in which three numbers in the range 1 through 16 are selected. Find the probability of winning this lottery, i.e., the probability of picking the three correct numbers. Then find the probability of picking exactly two of the three correct numbers. *(9 points)*

   Solution: In each case, we make a total of three choices total from the collection of 3 correct numbers and 13 incorrect ones (see the lottery handout for details). So $p(\text{all correct}) = \frac{{}_3C_3\cdot{}_{13}C_0}{{}_{16}C_3} = \frac{1}{560} \approx 0.0018$, and $p(\text{2 correct \& 1 wrong}) = \frac{{}_3C_2\cdot{}_{13}C_1}{=}\frac{3\cdot 13}{560} \approx 0.07$

4. If four dice are rolled, find the probability that they all show different numbers. Then find the probability that there is at least one repetition among the four. *(9 points)*

   Solution: The number of ways to choose 4 different numbers from 6 is ${}_6P_4$, and the number of ways to choose 4 numbers from 6 (allowing repitions) is 6^4, so $p(\text{all dice different}) = \frac{{}_6P_4}{6^4} = \frac{6\cdot5\cdot4\cdot3}{6\cdot6\cdot6\cdot6} \approx 0.28$ The probability of at least one repetition is computed using the probability rule for complementary events, so $p(\text{at least one repetition}) \approx 1 - 0.28 = 0.72$.

5. You and one of your math friends decide to play a game. Each of you rolls a die. If both dice are even, your friend pays you \$10. Otherwise, you pay your friend \$3. What is the expected value of this game from your point of view? Would this be a profitable game for you to play repeatedly? *(9 points)*

   Solution:

$$\text{Expected Value} = p(\text{both dice even})\cdot\$10 + p(\text{at least one die odd})\cdot(-\$3) = \frac{9}{36}\cdot\$10 - \frac{27}{36}\cdot\$3 = \$0.25$$

   So this would be a profitable game in the long run for you since you would win an average of 25 cents per game.

6. A fight is about to break out in the student cafeteria over whether math classes are more fun that statistics classes, or vice versa. A sociology major takes a quick survey and finds that among the men, 28 favor math, 21 favor statistics, and 8 have no preference, whereas among the women, 18 favor math, 23 favor statistics, and 4 have no preference. *(9 points)*

(a) Find the probability that a student prefers math.

Solution:

	Math	Stat	Neither	Totals
Men	28	21	8	57
Women	18	23	4	45
Totals	46	44	12	102

Make a table containing the relevant information. Then the number of students that prefer Math is 46, so $p(\text{a student prefers Math}) = \frac{46}{102} \approx 0.45$

(b) Find the probabiilty that a student prefers math, given that the student is a woman.

Solution: From the table, among women, we see that 18 out of 45 prefer math, so $p(\text{a student prefers math, given then student is a woman}) = \frac{18}{45} \approx 0.40$.

(c) Find the probability that a student is a women given that the student prefers math.

Solution: From the table, among students who prefer Math, we see that 18 out of 46 are women, so $p(\text{a student is a woman, given then student prefers math}) = \frac{18}{46} \approx 0.39$.

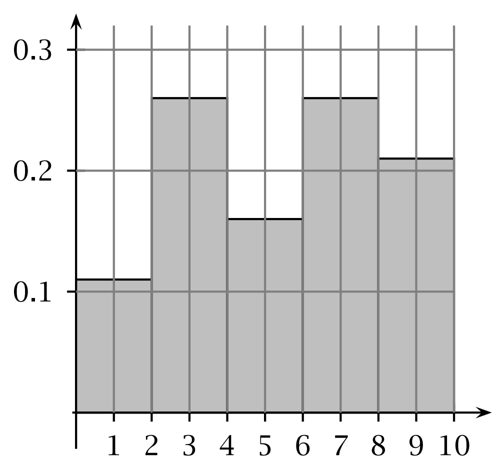
7. In a roll of two dice, let A be the event that the sum of the two dice is 6, and let B be the event that one of the dice is a 3. Are events A and B independent? Are they mutually exclusive? Explain. *(9 points)*

Solution: Since $p(A) = \frac{5}{36}$ is not equal to $p(A|B) = \frac{1}{11}$, A and B are not independent. Events A and B are also not mutually exclusive since they share the $(3,3)$ outcome.

8. Draw a *relative frequency histogram* for the dataset {5.1, 1.5, 2.3, 5.0, 9.1, 1.2, 7.4, 8.9, 7.1, 4.4, 3.2, 7.2, 8.7, 2.9, 3.1, 7.9, 2.1, 8.1, 7.9}. Use 5 data groups each of width 2 starting at 0 (so that $0 \le x < 2$ describes the first category). *(10 points)*

Solution:

category	freq.	relative freq.
$0 \le x < 2$	2	$\frac{2}{19} \approx 0.11$
$2 \le x < 4$	5	$\frac{5}{19} \approx 0.26$
$4 \le x < 6$	3	$\frac{3}{19} \approx 0.16$
$6 \le x < 8$	5	$\frac{5}{19} \approx 0.26$
$8 \le x \le 10$	4	$\frac{4}{19} \approx 0.21$
Total	19	



9. Calculate the mean and median of the data set $S = \{3, 1, 5, 3, 7, 4, 9, 11, 2, 8\}$ *(9 points)*

Solution: Arrange the numbers in order and take the average of the fifth and sixth values to get a median of 4.5. The mean is $\frac{53}{10} = 5.3$.

10. Find the variance and standard deviation for the data set $S = \{6, 4, 7, 9\}$. *(9 points)*

Solution: Since $\sum x = 26$ and $\sum x^2 = 182$ using the alternate formula for variance we get $s^2 = \frac{1}{n-1}\left[\sum x^2 - \frac{(\sum x)^2}{n}\right] = \frac{1}{3}\left[182 - \frac{676}{4}\right] \approx 4.33$. Then $s = \sqrt{s^2} = \sqrt{4.33} \approx 2.08$

11. If a population is normally distributed with mean 14.2 and standard deviation 2.8, find the probability that a randomly chosen member of the population is less than 12.0. *(9 points)*

Solution: $z_{12} = \frac{x-\bar{x}}{s} = \frac{12-14.2}{2.8} \approx -0.79$. From the table, $p(0 < z < .79) \approx .2148$, so $p(x < 12) = p(z < -.79) = 0.5 - .2852 = 0.2148$.