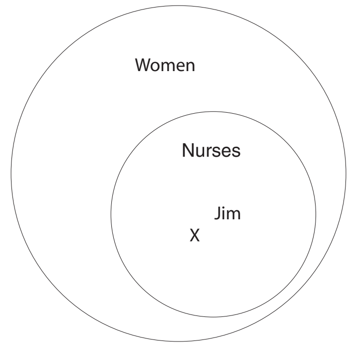
MA 110-07 §1.1 – 3.1	**Test #1**	*score*	**Name:** ______ **20 February 2001**

1. Use a properly labeled Venn diagram to determine the validity of the following argument. Explain. *(10 points)*

   1. All nurses are women.
   2. Bill is a nurse.

   Therefore Bill is a woman.



Solution: From the Venn diagram, we see that since Bill is a nurse and all nurses are women, Bill must indeed be a nurse.

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2. For the symbolic statement $p \to q$, construct a truth table to show that the converse of the statement is logically equivalent to the inverse of the statement. *(10 points)*

Solution: The converse of the statement is $q \to p$ and the inverse is ${\sim}p \to {\sim}q$.

p	q	${\sim}p$	${\sim}q$	$q \to p$	${\sim}p \to {\sim}q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	T	T

Since the truth tables are the same, the two statements are logically equivalent.

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3. Write the following argument in symbolic form. Then use a truth table to determine if the argument is valid. *(10 points)*

   If the defendant goes to jail, then the defendant is not innocent. If the defendant's lawyer is good, then the defendant does not go to jail. Therefore, the defendant is not innocent or the lawyer is not good.

Solution: Use the symbolic representations p: the defendant goes to jail; q: the defendant is innocent; r: the lawyer is good. Then the paragraph can be written as

$$\big((p \to {\sim}q) \wedge (r \to {\sim}p)\big) \to \big({\sim}q \vee {\sim}r\big)$$

which turns out to have one F in the truth table. So the argument is invalid.

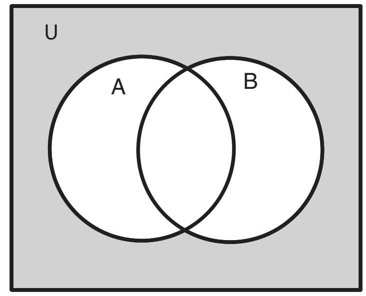
p	q	r	$p \to {\sim}q$	$r \to {\sim}p$	$p \to {\sim}q \wedge r \to {\sim}p$	${\sim}q \vee {\sim}r$	$\big((p \to {\sim}q) \wedge (r \to {\sim}p)\big) \to \big({\sim}q \vee {\sim}r\big)$
T	T	T	F	F	F	F	T
T	T	F	F	T	F	T	T
T	F	T	T	F	F	T	T
T	F	F	T	T	T	T	T
F	T	T	T	T	T	F	F
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

4. If $U = \{a, b, c, d, e, f, g, h, i, j, k, l, m\}$, $A = \{a, c, e, g\}$ and $B = \{g, i, k, m\}$, find the set $(A \cup B)'$. Illustrate $(A \cup B)'$ by shading the result in a Venn diagram. *(10 points)*

Solution:

Beginning with the universal set U, just delete all elements that are in $A \cap B$ to get $(A \cap B)' = \{b, d, f, h, j, l\}$.

The Venn diagram for $(A \cup B)'$ is shown to the right.



5. List all of the subsets of the set $\{a, b, c\}$. If A is a set with $n(A) = k$, how many subsets of A are there? Explain. *(10 points)*

   Solution: There are 8 subsets of the given set: $\{\}$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, and $\{a, b, c\}$. In general, a set with n elements has 2^n subsets. That is because the subsets of the given set can all be constructed as follows: for each of the n elements in the given set, decide whether or not it is admitted into the subset. That gives a total of n events, each with 2 outcomes – 2^n.

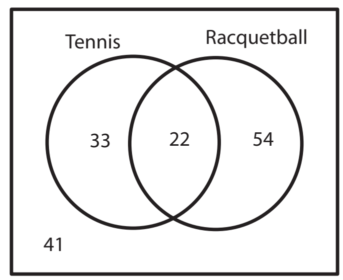
6. In a group of 150 students, 55 play tennis, 76 play racquetball, and 41 play neither? How many of the students play both tennis and racquetball?

   Draw a properly labeled Venn diagram and explain your reasoning. *(10 points)*

   Solution:

   Since 41 of the 150 students play neither, that means $n(T \cup R) = 150 - 41 = 109$. But we also know $n(T \cup R) = n(T) + n(R) - n(T \cap R)$, so $109 = 55 + 76 - n(T \cap R)$. Solving for the intersection, we see 22 students play both.

   The Venn diagram is shown to the right.



7. Compute the numbers ${}_6P_2$ and ${}_6C_2$. For the set $\{a, b, c, d, e, f\}$, list all the subsets of cardinality two and explain how this list is related to one of the numbers you just computed. *(10 points)*

   Solution:

   ${}_6P_2 = 6 \cdot 5 = 30$ and ${}_6C_2 = \frac{6\cdot 5}{2} = 15$. The number of subsets of cardinality 2 from a set of 6 elements is $-6C_2$: $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{a, e\}$, $\{a, f\}$, $\{b, c\}$, $\{b, d\}$, $\{b, e\}$, $\{b, f\}$, $\{c, d\}$, $\{c, e\}$, $\{c, f\}$, $\{d, e\}$, $\{d, f\}$, and $\{e, f\}$.

8. In how many ways can 6 math books and 4 stat books be lined up on a shelf (without regard to subject)? In how many ways can this be done if books of the same subject are adjacent? Explain your counting argument. *(10 points)*

   Solution: Ten books can be arranged on a sheld in $10! = 3,628,800$ ways.

   To keep the same subjects adjacent, we first pick an order for the two subjects (2 ways), the arrange the math books (6!) and finally arrange the stat books (4!). Taking the product, we get $2 \cdot 6! \cdot 4! = 34,560$.

9. From a group of 8 men and 9 women, how many different ways are there to choose a 5-person committee consisting of 3 women and 2 men? Explain your counting argument. *(10 points)*

   Solution: First we can choose 2 men from 8 (${}_8C_2$) and then choose 3 women from 9 (${}_9C_3$). Taking the product, we get ${}_8C_2 \cdot {}_9C_3 = 2,352$.

10. How many different 5-card poker hands consist of four of a kind? Explain. *(10 points)*

    Solution: First we select a kind (13), then select all four cards from the kind (1), then select a fifth card (48). Taking the product, we get $13 \cdot 48 = 624$.