MA 110-07 §3.1 – 4.3	**Test #2**	*score*	**Name:** _____ **17 April 2001**

1. Three coins are tossed. Write out the sample space S of all outcomes. Find the probability that 2 coins are heads and one is tails. Next, find the probability that exactly two of the three coins match. *(11 points)*

Sample Space

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

} 8 outcomes

$P(2H+1T) = \frac{3}{8} = .375$

$P(\text{exactly 2 match}) = \frac{6}{8} = .75$

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2. Find the probability of being dealt four-of-a-kind in a five-card hand. *(11 points)*

$n(4\text{-of-a-kind}) = 13 \cdot 1 \cdot 48 = 624$

choose a kind
get 4 cards from kind
choose 5th card

$P(4\text{-of-a-kind}) = \frac{624}{{}_{52}C_5} = .0002401$

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3. If two dice are rolled, find the probability that at least one is a 6. Then do the same problem, but with three dice, i.e., find the probability that at least one of the three dice is a 6. (HINT: consider the complementary event.) *(11 points)*

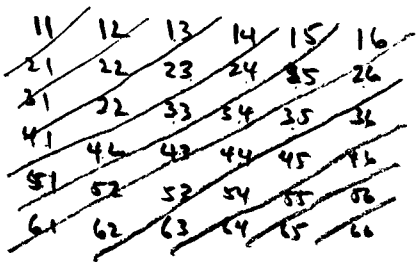
Look at the 6×6 table of outcomes for 2 dice & count. $\frac{11}{36} \approx .31$

For 3 dice, there are $6^3 = 216$ outcomes, so we count the complementary event

$P(\text{no six}) = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \frac{125}{216}$. So

$P(\text{at least one 6}) = 1 - \frac{125}{216} = .42$

4. Let's play a dice game where you roll two dice. If the sum is 5 or less, I pay you \$7. If the sum 6, 7, 8, or 9, you pay me \$5. If the sum is 10 or greater, I will again pay you \$7. Compute the expected value of this game (from your perspective). Would this be a profitable game for you to play repeatedly? *(11 points)*



$P(\text{sum} \le 5) = \frac{10}{36}$

$P(6 \le \text{sum} \le 9) = \frac{20}{36}$

$P(\text{sum} \ge 10) = \frac{6}{36}$

Expected Value $= \frac{10}{36}\cdot(7) + \frac{20}{36}(-5) + \frac{6}{36}(7) = \frac{112-100}{36} = \$0.33$

So the game would be profitable.

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5. A coin is flipped; then another coin is flipped. Let A denote the event that the first coin lands heads. Let B denote the event that exactly one coin lands heads. Are A and B mutually exclusive? Are A and B independent? Explain. *(11 points)*

Sample Space $= \{HH, HT, TH, TT\}$

$A \cap B = \{HT\} \neq \emptyset$, so A and B are <u>not</u> mutually exclusive.

$P(A|B) = \frac{1}{2}$ and $P(A) = \frac{2}{4} = \frac{1}{2}$, so A and B are independent (since $P(A|B) = P(A)$).

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6. Three coins are tossed. Find the probability that all three coins are heads given that at least two of the three coins are heads. *(11 points)*

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

These 4 outcomes fulfil the condition, so

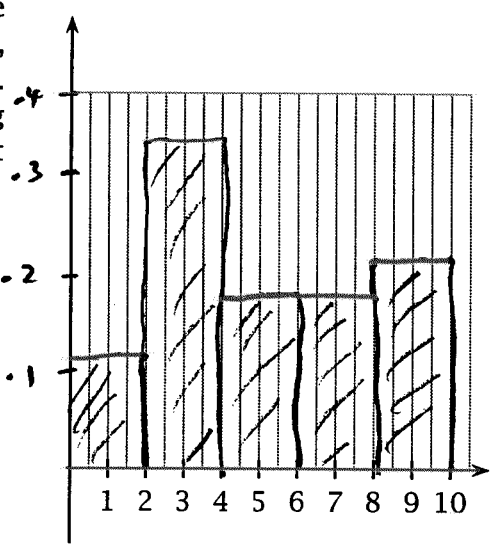
$P(\underbrace{\text{all are heads}}_{A} \mid \underbrace{\text{at least 2 are heads}}_{B}) = \frac{1}{4}$

Alternate way:

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/8}{4/8} = \frac{1}{4}$.

7. Draw a *relative frequence histogram* for the dataset {1.1, 1.5, 2.3, 5.0, 9.1, 3.2, 7.4, 3.9, 7.1, 3.2, 7.2, 8.6, 2.2, 3.7, 5.9, 5.1, 8.1, 9.9}. Use 5 data groups each of width 2 starting at 0 (so that $0 \le x < 2$ describes the first category). *(11 points)*

interval	tally	freq.	rel freq.
$0 \le x < 2$	II	2	$2/18 = .11$
$2 \le x < 4$	IIII I	6	$6/18 = .33$
$4 \le x < 6$	III	3	$3/18 = .17$
$6 \le x < 8$	III	3	$3/18 = .17$
$8 \le x < 10$	IIII	4	$4/18 = .22$
		18	



8. Calculate the mean, median, and mode of the data set $S = \{3, 2, 5, 3, 7, 4, 1, 4, 2, 5, 3, 6\}$ *(11 points)*

1 2 2 3 3 3 4 4 5 5 6 7

$$\text{mean} = \frac{45}{12} = 3.75$$

median = 3.5

mode = 3

9. Calculate the sample variance and sample standard deviation for the data set $S = \{22, 26, 21, 23\}$. *(11 points)*

$$\text{mean} = \frac{92}{4} = 23$$

$$\text{Variance} = \frac{(22-23)^2 + (26-23)^2 + (21-23)^2 + (23-23)^2}{3} = \frac{1+9+4+0}{3} = \frac{14}{3}$$

$$\approx 4.67$$

$$\text{Standard Deviation} = \sqrt{4.67} \approx 2.16$$