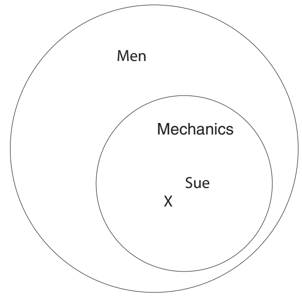
MA 110-91 §1.1 – 2.4	**Test #1 Solutions**	**Name:** ____________ **21 February 2002**

Test #1 Solutions

1. Use a properly labeled Venn diagram to determine the validity of the following argument. Explain. *(10 points)*

   1. All mechanics are men.
   2. Sue is a mechanic.

   Therefore, Sue is a man.

   Solution:

   From the Venn diagram, we see that since Sue is a mechanic and all mechanics are men, Sue must indeed be a mechanic. The argument is VALID.

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2. Construct a truth table to show that the that the statement $p \to q$ is logically equivalent to its contrapositive. *(10 points)*

   Solution: The contrapositive of the statement $p \to q$ is $\sim q \to \sim p$.

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Since the truth tables are the same, the two statements are logically equivalent.

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3. Write the following argument in symbolic form. Then use a complete truth table to determine if the argument is valid. *(10 points)*

   If the defendant goes to jail, then the defendant is not innocent. If the defendant's lawyer is good, then the defendant does not go to jail. Therefore, the defendant is innocent or the lawyer is good.

   Solution: Use the symbolic representations p: the defendant goes to jail; q: the defendant is innocent; r: the lawyer is good. Then the paragraph can be written as $\big((p \to \sim q) \wedge (r \to \sim p)\big) \to (q \vee r)$ which turns out to have one F in the truth table. So the argument is invalid.

p	q	r	$\sim p$	$\sim q$	$p \to \sim q$	$r \to \sim p$	$(p \to \sim q) \wedge (r \to \sim p)$	$q \vee r$	$\big((p \to \sim q) \wedge (r \to \sim p) \wedge \sim p\big) \to (q \vee r)$
T	T	T	F	F	F	F	F	T	T
T	T	F	F	F	F	T	F	T	T
T	F	T	F	T	T	F	F	T	T
T	F	F	F	T	T	T	T	F	T
F	T	T	T	F	T	T	T	T	F
F	T	F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	F	F

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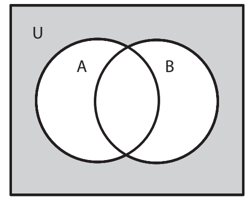
4. If $U = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n\}$, $A = \{a, d, g, j, m\}$ and $B = \{a, c, e, g, i, k, m\}$, enumerate the set $(A \cup B)'$. Then illustrate $(A \cup B)'$ by shading the result in a Venn diagram. Then illustrate $(A \cup B)'$ by shading the result in a Venn diagram. *(10 points)*

   Solution: The set $(A \cup B)'$ consists of all the elements that are not in $A \cup B$. So, $(A \cup B)' = \{b, f, h, l, n\}$.

   The Venn diagram for $(A \cup B)'$ is shown to the right.

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5. List all the subsets of $\{a, b\}$. Then list all the subsets of the set $\{a, b, c\}$. *(10 points)*

Solution: There are 4 subsets of $\{a, b\}$: $\emptyset$, $\{a\}$, $\{b\}$, $\{a, b\}$.

There are 8 subsets of $\{a, b, c\}$: $\{\}$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, and $\{a, b, c\}$.

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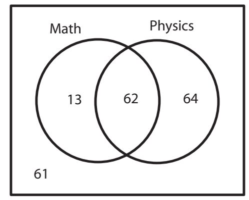
6. In a group of 200 students, 75 play tennis, 126 play racquetball, and 61 play neither. How many of the students play both tennis and racquetball?

Draw a properly labeled Venn diagram and explain your reasoning. *(10 points)*

Solution:

Since 61 of the 200 students play neither, that means $n(M \cup P) = 200 - 61 = 139$. But we also know $n(M \cup P) = n(M) + n(P) - n(M \cap P)$, so $139 = 75 + 126 - n(T \cap R)$. Solving for the intersection, we see 62 students enjoy both.

The Venn diagram is shown to the right.



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7. Compute the numbers ${}_5P_2$ and ${}_5C_2$. For the set $\{a, b, c, d, e\}$, list all the subsets of cardinality two and explain how this list is related to one of the numbers you just computed. *(10 points)*

Solution: ${}_5P_2 = 5 \cdot 4 = 20$, and ${}_5C_2 = 5 \cdot 4/2 = 10$. The subset of cardinality 2 of $\{a, b, c, d, e\}$ are $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{a, e\}$, $\{b, c\}$, $\{b, d\}$, $\{b, e\}$, $\{c, d\}$, $\{c, e\}$, $\{d, e\}$. There are 10 such subsets since each one corresponds to an selection without regard to order of 2 elements from the 5. That is exactly what ${}_5C_2$ measures.

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8. In how many ways can 5 girls and 4 boys be lined up in a row (without regard to gender)? In how many ways can this be done if children of the same gender are adjacent? Explain your counting argument. *(10 points)*

Solution: Nine children can be arranged in a row in $9! = 362,880$ ways.

To keep children of the same gender adjacent, we first pick an order for the two genders (2! ways), then arrange the boys (4!), then the girls (5!). Taking the product, we get $2! \cdot 4! \cdot 5! = 5,760$.

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9. From a group of 7 men and 10 women, how many different ways are there to choose a 5-person committee consisting of 3 women and 2 men? Explain your counting argument. *(10 points)*

Solution: First we can choose 2 men from 7 (${}_7C_2$) and then choose 3 women from 10 (${}_{10}C_3$). Taking the product, we get ${}_7C_2 \cdot {}_{10}C_3 = 2,520$.

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10. From a standard 52-card deck, how many different 5-card hands consist of four cards in one suit with the fifth card in a different suit? Explain. *(10 points)*

Solution: First we select a suit (4 ways), then four cards from the selected suit (${}_{13}C_4$), and finally one remaining card (39). Taking the product, we get $4 \cdot {}_{13}C_4 \cdot 39 = 111,540$.