## Matching Numbers in a Lottery

The number of ways of selecting 6 numbers from the numbers 1 through 49 (without replacement) is

$$
{ }_{49} C_{6}=\frac{49!}{6!(49-6)!}=13,983,816
$$

Now supose that 6 numbers have been selected as above by Player A. The number of ways that Player B may choose 6 numbers between 1 and 49 (without repetition) so that exactly one of the chosen numbers matches one of the six selected by Player A is

$$
{ }_{6} C_{1} \cdot{ }_{43} C_{5}=6 \cdot 962,598=5,775,588
$$

The first factor is the number of ways Player B can select one number from the 6 selected by Player A. The second factor is the number of ways the 5 remaining numbers can be selected by Player B so that none matches the remaining 5 numbers chosen by Player A.

Similarly, the number of ways exactly two of the six numbers chosen by Player B match those selected by Player A is

$$
{ }_{6} C_{2} \cdot{ }_{43} C_{4}=15 \cdot 123,410=1,851,150
$$

The number of ways there are exactly three matches is

$$
{ }_{6} C_{3} \cdot{ }_{43} C_{3}=20 \cdot 12,341=246,820
$$

The mumber of ways there are exactly four matches is

$$
{ }_{6} C_{4} \cdot{ }_{43} C_{2}=15 \cdot 903=13,545
$$

The number of ways there are exactly five matches is

$$
{ }_{6} C_{5} \cdot{ }_{43} C_{1}=6 \cdot 43=258
$$

And, of course, the number of ways all six match is 1 .
To compute probabilities of of the above events, just take quotients of the number of occurances of the events with ${ }_{49} C_{6}$. For example, the probability of choosing exactly one correct number is

$$
\frac{5,775,588}{13,983,816} \approx 0.413019
$$

EXERCISE: Calculate the probabilities of choosing two correct numbers, three, etc., in this lottery.

