MA 110-02 §1.1 – 2.4	**Test #1**	*score*	**Name:** ________ **26 June 2001**

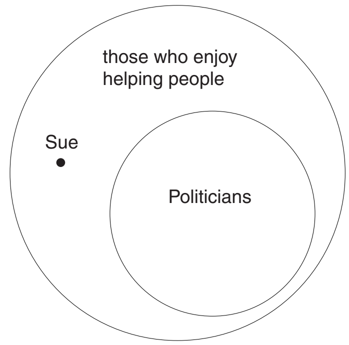
1. Use a properly labeled Venn diagram to determine the validity of the following argument. Explain. *(10 points)*

   1. All politicians enjoy helping people.
   2. Sue enjoys helping people.

   Therefore Sue is a politician.

   Solution:

   The argument is invalid. From the Venn diagram, we see Sue can enjoy helping while not being a politician.



2. Construct a truth table to show that the symbolic statement $p \to q$ is logically equivalent to its contrapositive. *(10 points)*

   Solution:

p	q	$p \to q$	$\sim q \to \sim p$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

The last two columns agree, so the statement $p \to q$ is logically equivalent to $\sim q \to \sim p$.

3. Write the following argument in symbolic form. Then use a truth table to determine if the argument is valid. *(10 points)*

   If a student studies regularly, then the student does well in school. If the student's teachers are good, then the student does well in school. The student does not do well in school. Therefore, the student doesn't study regularly or the student's teachers are not good.

   Solution:

   Use the symbolic representations p: a student studies regularly; q: the student does well in school; r: the student's teachers are good. Then the paragraph can be written as

$$\big((p \to q) \wedge (r \to q) \wedge (\sim q)\big) \to \big(\sim p \vee \sim r\big)$$

which turns out to be a tautology, so the argument is valid.

p	q	r	$p \to q$	$r \to q$	$\sim q$	$(p \to q) \wedge (r \to q) \wedge (\sim q)$	$\sim p \vee \sim r$	paragraph
T	T	T	T	T	F	F	F	T
T	T	F	T	T	T	T	T	T
T	F	T	F	F	F	F	F	T
T	F	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	F	F	F	T	T
F	F	F	T	T	T	T	T	T

4. Which two of the following statements are logically equivalent? (You don't need to use a truth table, but explain why they are in a sentence.) *(10 points)*

   (a) If it is not raining, then I play tennis.
   (b) If I play tennis, then it is not raining.
   (c) If it is raining, then I don't play tennis.
   (d) I hate tennis, therefore I don't play tennis.

   Solution:

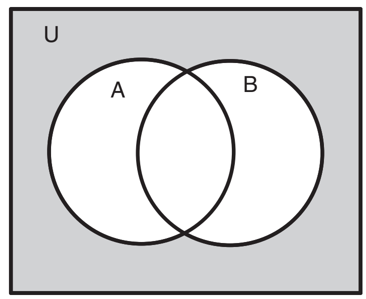
   Statement (c) is the contrapositive of statement (b), so they are logically equivalent. Statement (b) is the converse of (a) so they aren't equivalent.

5. If $U = \{a, b, c, d, e, f, g, h, i, j, k, l, m\}$, $A = \{a, c, d, g, j, k, m\}$ and $B = \{a, c, e, g, i, k, m\}$, find the set $(A \cap B)'$. Then illustrate $(A \cup B)'$ by shading the result in a Venn diagram. *(10 points)*

   Solution:

   Beginning with the universal set U, just delete all elements that are in $A \cap B$ to get $(A \cap B)' = \{b, d, e, f, g, h, i, l\}$.

   The Venn diagram for $(A \cup B)'$ is shown to the right.

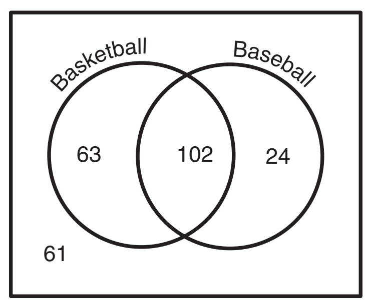


6. In a group of 250 students, 165 enjoy attending basketball games, 126 enjoy attending baseball games, and 61 enjoy neither? How many of the students enjoy both?

   Draw a properly labeled Venn diagram and explain your reasoning. *(10 points)*

   Solution:

   We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Since we are given that 61 like neither, we know that $250 - 61 = 189$ like at least one, so $n(A \cup B) = 189$. So $189 = 164 + 128 - n(A \cap B)$. Thus, $n(A \cap B) = 102$.



7. Compute the numbers ${}_7P_3$ and ${}_7C_3$. Make up two counting problems that would have these numbers as an answer. *(10 points)*

   Solution:

   Using the formulas we have ${}_7P_3 = \frac{7!}{(7-3)!} = 7 \cdot 6 \cdot 5 = 210$ and ${}_7C_3 = \frac{7!}{(7-3)!3!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2} = 35$. For counting problems, you could say:

   From a group of 7 people, we want to select 3 and line them up in a row (answer ${}_7P_3$).
   From a group of 7 people, we want to select 3 to form a committee (answer ${}_7C_3$).

8. In how many ways can 6 girls and 4 boys be lined up in a row (without regard to gender)? In how many ways can this be done if children of the same gender are adjacent? Explain your counting argument. *(10 points)*

   Solution:

   For the first part, we just line up 10 people: ${}_{10}P_{10} = 10! = 3,628,800$. For the second part, we can count it b performing the following 3 events:

   (a) Decide if the boys are first or the girls (2).
   (b) Arrange the boys (4!).
   (c) Arrange the girls (6!).

   Then the total number of ways to arrange the children is $2 \cdot 4! \cdot 6! = 34,560$.

9. From a group of 7 men and 9 women, how many different ways are there to choose a 5-person committee consisting of 3 women and 2 men? Explain your counting argument. *(10 points)*

   Solution:

   Perform the following 2 events to construct the committee:

   (a) Select 3 women from 9 (${}_9C_3$).
   (b) Select 2 men from 7 (${}_7C_2$).

   Multiply the results to get ${}_9C_3 \cdot {}_7C_2 = \frac{9!}{6! \cdot 3!} \cdot \frac{7!}{5! \cdot 2!} = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2} \cdot \frac{7 \cdot 6}{3 \cdot 2} = 84 \cdot 21 = 1,764$

10. How many different 5-card hands have exactly three hearts? Explain. *(10 points)*

    Solution:

    Perform the following 2 events to construct the hand:

    (a) Select 3 hearts from the 13 hearts in a deck (${}_{13}C_3$).
    (b) Select the 2 other cards needed to complete the hand from the 39 non-hearts in a deck (${}_{39}C_2$).

    Multiply the results to get ${}_{13}C_3 \cdot {}_{39}C_2 = \frac{13!}{10! \cdot 3!} \cdot \frac{39!}{37! \cdot 2!} = \frac{13 \cdot 12 \cdot 11}{3 \cdot 2} \cdot \frac{39 \cdot 38}{3 \cdot 2} = 286 \cdot 741 = 211,926$