

1. Two dice are rolled. Find the probability that the sum of the dice is 9 or larger. Explain. (11 points)

Write out the 36 outcomes and count those with a sum of 9 or larger

| | | | | | |
|----|----|----|----|----|----|
| 11 | 12 | 13 | 14 | 15 | 16 |
| 21 | 22 | 23 | 24 | 25 | 26 |
| 31 | 32 | 33 | 34 | 35 | 36 |
| 41 | 42 | 43 | 44 | 45 | 46 |
| 51 | 52 | 53 | 54 | 55 | 56 |
| 61 | 62 | 63 | 64 | 65 | 66 |

$$p = \frac{10}{36} \approx .28$$

2. Find the probability of being dealt a flush (all cards in the same suit) in a five-card hand. Explain. (11 points)

$$p(\text{flush}) = \frac{4 \times {}_{13}C_5}{{}_{52}C_5} \approx 0.002$$

3. If three dice are rolled, find the probability that at least one of the dice is a six. Explain. (11 points)

$$\begin{aligned} p(\text{at least one is a 6}) &= 1 - p(\text{none is a 6}) \\ &= 1 - \frac{5^3}{6^3} \approx .42 \end{aligned}$$

4. Player A selects a card from a standard 52-card deck. If the card is a 10 or below (Ace counts low), Player A wins \$10 from Player B. If Player A selects a Jack, Queen, or King, Player A pays Player B \$30. Find the expected value of the game for Player A. (11 points)

$$\begin{aligned} \text{Expected value} &= p(\text{card} \leq 10) \cdot (\$10) + p(J, Q, K) \cdot (-\$30) \\ &= \frac{10}{13} (10) + \frac{3}{13} (-30) = \frac{100 - 90}{13} = \frac{10}{13} \approx \$0.77. \end{aligned}$$

5. A die is rolled, then another die is rolled. Let A denote the event that the sum of the dice is 6. Let B denote the event that the second die is a 5. Are events A and B independent? Are they mutually exclusive? Explain. (11 points)

$$P(A) = \frac{5}{36}$$

$$P(A|B) = \frac{1}{6}$$

So A and B are not independent.

They are not mutually exclusive, since they have the (1,5) outcome in common.

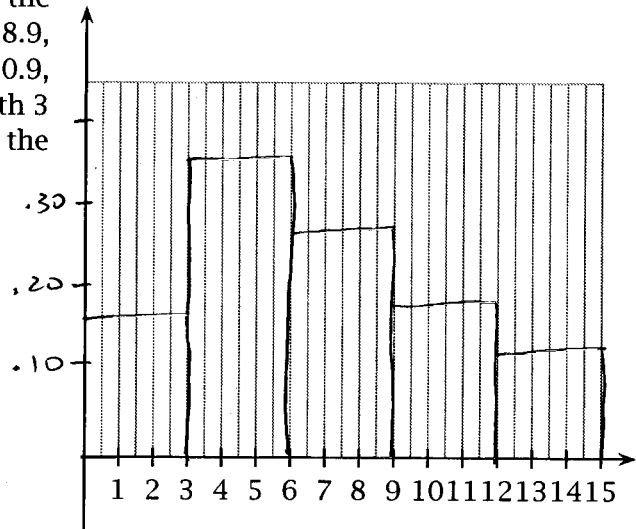
6. In a survey of 200 men and 300 women, we find that 175 of the men like to watch football and 25 do not, whereas, 160 of the women like to watch football and 140 do not. Find the probability that a randomly selected person in the survey likes to watch football. Then find the probability that a person is a man, given that the person likes to watch football. (11 points)

$$P(\text{like football}) = \frac{175 + 160}{500} = \frac{335}{500} = .67$$

$$P(\text{man} | \text{likes football}) = \frac{175}{175 + 160} = \frac{175}{335} \approx .74$$

7. Draw a *relative frequency histogram* for the dataset {5.1, 2.5, 12.3, 9.1, 7.1, 4.1, 13.4, 8.9, 3.1, 5.2, 6.8, 3.6, 2.2, 6.7, 3.9, 11.7, 7.1, 10.9, 1.2, 11.1}. Use 5 data groups each of width 3 starting at 0 (so that $0 \leq x < 3$ describes the first category). (12 points)

| x | freq | | rel. freq |
|---------------------|------|----|----------------------|
| $0 \leq x < 3$ | | 3 | $\frac{3}{20} = .15$ |
| $3 \leq x < 6$ | | 7 | $\frac{7}{20} = .35$ |
| $6 \leq x < 9$ | | 5 | $\frac{5}{20} = .25$ |
| $9 \leq x < 12$ | | 3 | $\frac{3}{20} = .15$ |
| $12 \leq x \leq 15$ | | 2 | $\frac{2}{20} = .10$ |
| total | | 20 | |



8. Calculate the mean, sample variance, and sample standard deviation of the sample data set $S = \{6, 2, 5, 7\}$ (11 points)

$$\bar{x} = \frac{20}{4} = 5$$

$$s^2 = \frac{1^2 + 3^2 + 0^2 + 2^2}{3} = \frac{14}{3} \approx 4.67$$

$$s \approx 2.16$$

9. A population is normally distributed with mean 42 and standard deviation 4. Find the probability that a randomly selected member of the population is between 41 and 45. (Refer to the table on the next page as needed.) (11 points)

$$z_{41} = \frac{41 - 42}{4} = -0.25$$

$$z_{45} = \frac{45 - 42}{4} = 0.75$$

$$\begin{aligned} P(41 \leq x \leq 45) &= P(-0.25 \leq z \leq 0.75) = P(0 \leq z \leq 0.25) + P(0 \leq z \leq 0.75) \\ &= 0.0987 + 0.2734 \\ &= 0.3721 \end{aligned}$$