MA 110-11 §3.1 – 4.3	**Test #2**	*score*	**Name:** _____ **19 June 2002**

1. A couple plans to have four children. What is the probability they will have exactly two boys and two girls? Explain. *(9 points)*

   Solution: $p(\text{2 boys and 2 girls}) = \frac{{}_4C_2}{2^4} = \frac{6}{16} = 0.375$

2. You select three cards from a standard 52-card deck. What is the probability that all three cards are in the same suit? Explain. *(9 points)*

   Solution: $p(\text{3 cards in same suit}) = \frac{{}_4C_1 \cdot {}_{13}C_3}{{}_{52}C_3} = \frac{4 \cdot 286}{22,100} \approx 0.051765$

3. You roll three standard dice. What is the probability that the roll results in three different numbers, i.e., that there are are no matches. *(9 points)*

   Solution: $p(\text{no match} = \frac{{}_6P_3}{6^3} = \frac{6 \cdot 5 \cdot 4}{6 \cdot 6 \cdot} = \frac{5}{9} \approx 0.56$

4. We play a lottery in which three numbers in the range 1 through 14 are selected. Find the probability of winning this lottery, i.e., the probability of picking the three correct numbers. Then find the probability of picking exactly two of the three correct numbers. Explain. *(9 points)*

   Solution: $p(\text{all three numbers}) = \frac{{}_3C_3}{{}_{14}C_3} = \frac{1}{364} \approx 0.003$. $p(\text{exactly two match}) = \frac{{}_3C_2 \cdot {}_{11}C_1}{{}_{14}C_3} = \frac{3 \cdot 11}{364} \approx 0.09$

5. You and one of your MA 110 friends play a game. You draw a single card from a standard 52-card deck. If the card is an Ace through five, your friend pays you \$1. If the card is a six through ten, your friend pays you \$2. If the card is a face card (Jack, Queen, or King) you pay your friend \$4. Calculate the expected value of the game from your point of view. Whom does the game favor? *(9 points)*

   Solution: $E = p(\text{Ace through 5}) \cdot \$1 + p(\text{6 through 10}) \cdot \$2 + p(\text{face card}) \cdot (-\$4) = \frac{5}{13} + \frac{5}{13}2 + \frac{3}{13}(-4) = \$0.23$. So the game favors you.

6. A red die and a blue die are rolled simultaneously. Let E_1 denote the event that the red die is a 6, and let E_2 denote the event that the sum of the two dice is 7. Are E_1 and E_2 mutually exclusive? Explain. Are they independent? Explain. *(9 points)*

   Solution: The outcome in which the red die is 6 and the blue die is 1 is common to both E_1 and E_2, so those events are not mutually exclusive. Also, since $p(E_2) = \frac{1}{6}$ and $p(E_2|E_1) = \frac{1}{6}$, the events are independent.

7. If a computer breaks down on a space mission with probability 0.02, and if we send a total of three identical computers on the mission, what is the probability that at least one is working for the entire mission? *(9 points)*

   Solution: If we assume that the breakdown of the computers is independent, the probability of all three breaking down is $(0.02)^3$. Thus, $p(\text{at least one working}) = 1 - (.02)^3 = 0.999998$.

8. A group of Finite Math students conducts a survey on barbeque sauces in order to prepare for the annual Fourth of July Finite Math Cookout. Individuals are asked if they prefer Bubba's sauce or Billy Bob's sauce. The results are summarized below. *(9 points)*

	Bubba's	*Billy Bob's*	*no opinion*
Men	195	188	35
Women	158	184	13

(a) Find the probability that a random person prefers Bubba's.

Solution:

$$p(\text{Bubba's}) = \frac{353}{773} \approx 0.46$$

(b) Find the probability that a person prefers Bubba's given that the person is a woman.

Solution:

$$p(\text{Bubba's}|\text{woman}) = \frac{158}{353} \approx 0.45$$

(c) Find the probability that a person is a woman given that the person prefers Billy Bob's sauce.

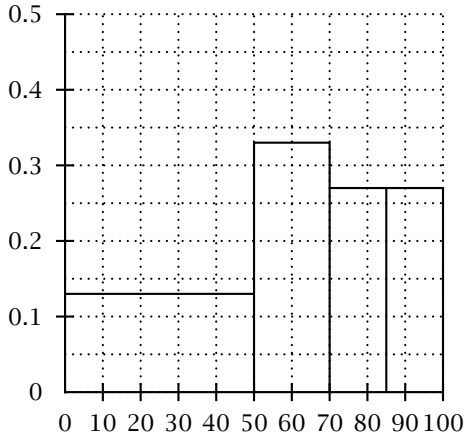
Solution:

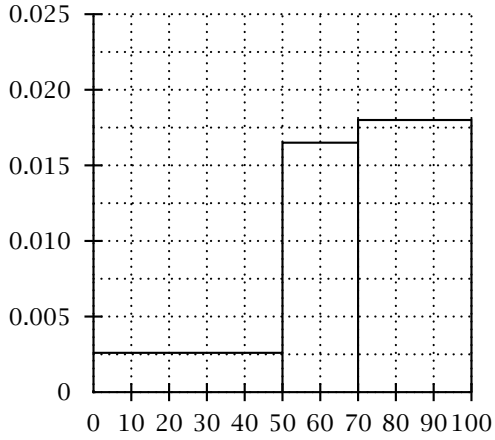
$$p(\text{woman}|\text{Bubba's}) = \frac{184}{355} \approx 0.52$$

9. Construct relative frequency and relative frequency density histograms on the axes below for the dataset $S = \{25, 98, 55, 46, 64, 89, 75, 83, 76, 58, 64, 86, 55, 84, 87\}$ using the four intervals $0 \le x < 50$, $50 \le x < 70$, $70 \le x < 85$, and $85 \le x \le 100$. *(10 points)*

Solution:

interval	*frequency*	*rf*	*rfd*
$0 \le x < 50$	2	$\frac{2}{15} \approx 0.13$	$\frac{2}{15\cdot 50} \approx 0.003$
$50 \le x < 70$	5	$\frac{5}{15} \approx 0.33$	$\frac{5}{15\cdot 20} \approx 0.017$
$70 \le x < 85$	4	$\frac{4}{15} \approx 0.27$	$\frac{4}{15\cdot 15} \approx 0.018$
$85 \le x \le 100$	4	$\frac{4}{15} \approx 0.27$	$\frac{4}{15\cdot 15} \approx 0.018$





10. Calculate the mean and median of the data set $S = \{8, 3, 12, 17, 11, 14, 9, 7, 5, 13\}$ *(9 points)*

Solution:

$$\bar{x} = \frac{99}{10} = 9.9$$

Median=10 (the average of the data 9 and 11)

11. Calculate the sample variance and sample standard deviation of the dataset $S = \{5, 8, 6, 10\}$. *(9 points)*

Solution:

$$s^2 = \frac{1}{n-1}\left[\sum x^2 - \frac{(\sum x)^2}{n}\right] = \frac{1}{3}\left[225 - \frac{841}{4}\right] \approx 4.92$$

$$s = \sqrt{s^2} \approx \sqrt{4.92} \approx 2.22$$