Name: $\qquad$
score 20 June 2002

1. Let $f(x)=x e^{-x}$ on the interval $[-2,4]$. Find the global maximum and minimum for $f$ on this interval. Do this by first finding all the critical point for $f$ on the interval. Use exact algebraic calculations as much as possible, approximating on a calculator only when required. Then sketch the graph of $f$ on this interval and label the critical point(s) and the global extrema. (Use your graphing calculator to assist with this.) (10 points)
2. The graph indicates the cost $C$ (in dollars along the vertical axis) of manufacturing $q$ items (along the horizontal axis). Suppose that 600 items are being made. Assuming that all items manufactured can be sold, should the production be increased or decreased to decrease average cost. Explain. Then approximate the value of $q$ graphically that minimizes the average cost. (10 points)

3. The table shown below gives the velocity $v(t)$ of an object (in feet/second) at time $t$ (in seconds). Using the only the given data, find upper and lower estimates of the total distance covered by the object from 0 to 8 seconds. Next, notice that the equation $v(t)=20+\frac{1}{2} t^{2}$ correctly reproduces the given data. Then calculate the exact distance travelled using this velocity function. How do your various answers compare? (10 points)

| $t$ | 0 | 2 | 4 | 6 | 8 |
| :---: | ---: | ---: | ---: | ---: | ---: |
| $v(t)$ | 20 | 22 | 28 | 38 | 52 |

4. Suppose the demand for a product is given by the equation $q=10000-e^{0.2 p}$ where $q$ is the quantity in units and $p$ is the price in dollars. Determine the elasticity when the price is $\$ 4$ and again when the price is $\$ 6$. At each price, explain whether you should increase the price or decrease the price in order to increase revenue? (10 points)
5. A small area in the first quadrant is bounded by the two curves $y=-x^{3}+2 x^{2}$ and $y=x^{2}$. Sketch a graph of the region and calculate its area (approximately or exactly). (10 points)
6. Find the average value of the function $f(x)=x^{2}-6 x+10$ on the interval [ 0,7$]$. Sketch a graph of $f$ and illustrate the average value graphically to see if you answer makes intuitive sense. (10 points)
7. For the supply and demand curve shown, estimate the equilibrium price and quantity. Price is in dollars per unit on the vertical axis and quantity is in thousands along the horizontal axis. Then estimate the consumer and producer surplus at the equilibrium price. Shade in and label the areas on the graph that you are using for the surpluses. If the price is changed to $\$ 20$, how does this impact the consumer and producer surplus? (10 points)

8. A company estimates it will have a continuous revenue stream in dollars per year over the next 5 years to be $R(t)=100000(1+0.2 t)$. Find the present value of the 5 -year revenue stream. (10 points)
9. Find the general anti-derivative of the function $f(x)=x^{2}-8 x+10$. Then use this antiderivative to calculate the exact value of $\int_{0}^{4} f(x) d x$. Sketch a graph of $f$ on this interval and explain geometrically what your answer means in terms of areas. (10 points)
10. Sketch the graph of $f(x)$ on the axes provided given that $f(0)=-1$. The graph shown is that of $f^{\prime}(x)$. (10 points)

