MA 126-02 §8.1–8.8	**Test #2**	*score*	**Name:** ________________ **21 July 2000**

1. For each sequence, determine if it converges (and find the limit) or diverges. Fully explain your work. *(8 points each)*

   (a) $a_n = \frac{1}{n} \ln \frac{1}{n}$

   (b) $a_1 = 1$, $a_{n+1} = 1 - a_n$ for $n > 1$

2. If the n^{th} partial sum of a series $\sum_{n=1}^{\infty} a_n$ is given by $s_n = \frac{2n}{n+1}$, find a_n and the sum $\sum_{n=1}^{\infty} a_n$. *(8 points)*

3. Determine if the series $\displaystyle\sum_{n=0}^{\infty} \frac{1 + (-5)^n}{2^{3n}}$ converges or diverges. If it converges, determine its sum. Explain. *(8 points)*

4. For each of the following series, determine if it converges absolutely, converges conditionally, or diverges. Explain your reasoning. *(8 points each)*

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n^3 - n + 1}$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{k^2}{2^k}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

5. Determine the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{2^n}{(n+1)3^n} x^n$. *(9 points)*

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6. The hyperbolic cosine function is defined as $\cosh x = \frac{e^x + e^{-x}}{2}$. Find the Maclaurin series for $\cosh x$ beginning with the series for e^x. *(9 points)*

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7. Show that $\frac{d}{dx} \sin x = \cos x$ by beginning with the Maclaurin series for $\sin x$, differentiating term by term, and showing that the result is the Maclaurin series for $\cos x$. *(9 points)*

8. In order to approximate $e^{0.1}$ using the Maclaurin series for e^x, what order Maclaurin polynomial would be required to guarantee that the result is within 10^{-6} of the correct value? Write out the Maclaurin polynomial you would use. *(9 points)*

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9. Explain how the binomial series could be used to approximate $\sqrt[3]{1001}$ by writing down the first four terms in the appropriate series. Then compute the sum of those four terms and give your approximation in decimal form. How many decimal places of accuracy do you think the answer has. Why? *(8 points)*