Review Problems for Test #2

MA 126 — Summer 2000

1. Determine if the following sequences converge or diverge. If they converge, find the limit.

   (a) $a_n = \dfrac{n^2 - 1}{n^2 + 1}$

   (b) $a_n = \left(1 + \dfrac{1}{n}\right)^n$

   (c) $a_n = \dfrac{\cos^2 n}{n}$

   (d) $a_n = \dfrac{n! - 1}{n!}$

2. Use the integral test to determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n \ln n}$ converges.

3. Determine if the following series converge or diverge. Explain.

   (a) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n}$

4. Determine if the following series converge absolutely, converge conditionally, or diverge. Explain fully.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

   (b) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{n \ln n}{2^n}$

5. Determine the interval of convergence for the power series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}\, x^n}{3^n}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$

6. Write the power series representation for the function $\dfrac{1}{1+x}$ and state its interval of convergence. Differentiate this power series term by term to deduce the power series representation for the function $\dfrac{1}{(1+x)^2}$.

7. Beginning with the Maclaurin series for $\cos x$, find the Maclaurin series for $x \cos(2x)$.

8. The hyperbolic sine function is defined as $\sinh x = \dfrac{e^x - e^{-x}}{2}$. Find the Maclaurin series for $\sinh x$.

9. How many terms of the Maclaurin series for e^x would you have to sum in order for the partial sum to approximate the value of e^1 correct to 8 decimal places?

10. Use the binomial series to find the Maclaurin series for $\sqrt{4+x}$ and determine the radius of convergence. Then use a partial sum of the series to estimate the value of $\sqrt{4.1}$ correct to 4 decimal places. Explain how you know how many terms in the series to use.