MA 227 §12.3–12.5, §12.7	**Quiz #5**	*score*	**Name:** ______________ **19 July 1999**

1. Evaluate the integral by reversing the order of integration. *(5 points)*

$$\int_0^1 \int_y^1 \sin(x^2)\, dx\, dy$$

$\int_0^1 \int_y^1 \sin(x^2)\, dx\, dy = \int_0^1 \int_0^x \sin x^2\, dy\, dx = \int_0^1 x \sin x^2\, dx = -\frac{1}{2}\cos x^2 \Big|_0^1 =$
$-\frac{1}{2}\cos 1 + \frac{1}{2}$

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2. Use double integrals to find the volume of the solid bounded above by the cone $z = 4 - \sqrt{x^2 + y^2}$ and below by the xy-plane. *(5 points)*

$\iint_R \left(4 - \sqrt{x^2 + y^2}\right) dA = \int_0^{2\pi} \int_0^4 (4 - r) r\, dr\, d\theta = \int_0^{2\pi} \left(4r - \frac{1}{2}r^2\right) \Big|_0^4 d\theta =$
$2\pi\left(32 - \frac{64}{3}\right) = \frac{64\pi}{3}$

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3. Find the center of mass of the lamina bounded by the x-axis and the curve $y = \cos^2 x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ if the mass density function is $\rho(x, y) = y$. *(6 points)*

$M = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\cos^2 x} y\, dy\, dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}\cos^4 x\, dx = \int_0^{\frac{\pi}{2}} \cos^4 x\, dx = \frac{3}{4\cdot 2} \cdot \frac{\pi}{2} = \frac{3\pi}{16}$
$M_x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\cos^2 x} y^2\, dy\, dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{3}\cos^6 x\, dx = \frac{2}{3}\int_0^{\frac{\pi}{2}} \cos^6 x\, dx = \frac{2}{3}\frac{5\cdot 3}{6\cdot 4\cdot 2} \cdot \frac{\pi}{2} = \frac{5\pi}{48}$
$M_y = 0$ by symmetry, thus
$(\bar{x}, \bar{y}) = \left(0, \frac{5}{9}\right)$

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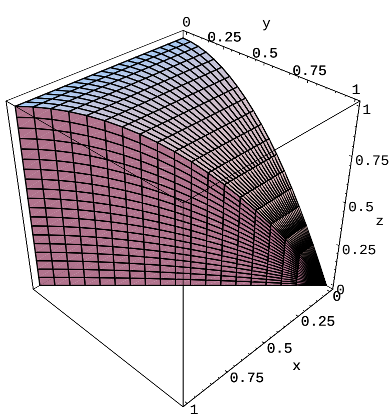
4. Set up (but do not evaluate) the following triple integral as an iterated integral where S is the solid located in the first octant bounded by the coordinate planes, the surface $z = 1 - y^2$ and the plane $x + y = 1$. *(4 points)*

$$\iiint_S f(x, y, z)\, dV$$



There are many ways to set up the integration, one of which is

$$\int_0^1 \int_0^{1-x} \int_0^{1-y^2} f(x, y, z)\, dz\, dy\, dx$$