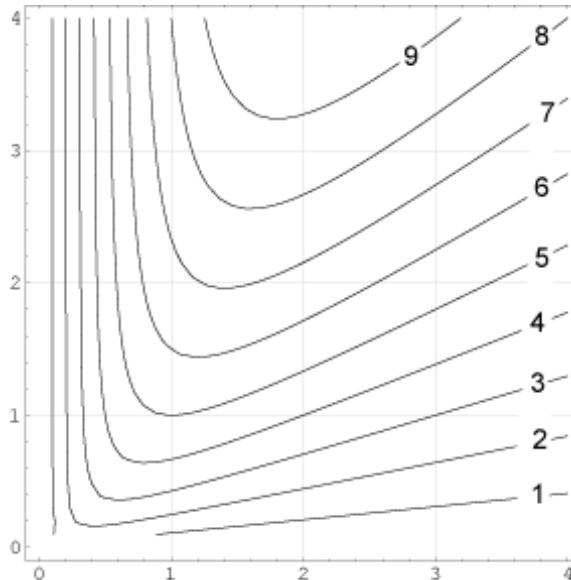


1. Suppose the level curves for a function $f(x, y)$ are as given. Use the level curve diagram to do the following.

- (a) Estimate the values of $f_x(1, 1)$ and $f_y(1, 1)$. (10 points)
- (b) Estimate the value of $f_u(2, 1)$ where u is a unit vector making an angle of $-\frac{\pi}{4}$ with the positive x -axis. (10 points)
- (c) Draw the gradient vector at the point $(2, 2)$, $\nabla f(2, 2)$ on the level curve diagram. Place the tail of the vector at $(2, 2)$. (10 points)



2. Let $f(x, y) = \sqrt{x^2 + y^4}$. Compute the differential of f and simplify the result. (12 points)
3. A right circular cylinder is changing its radius and height with respect to time. Determine the rate of change of the volume of the cylinder when the radius is 3 inches and the height is 5 inches if the radius is increasing at 2 in/sec and the height is decreasing at 3 in/sec. (11 points)
4. Find the second degree Taylor polynomial about the origin of the function $f(x, y) = \sin(xy)$. From this Taylor polynomial, conclude that the origin is a critical point of f and use the Taylor polynomial to determine which type of critical point you have found. (12 points)
5. Find all critical points for the function $f(x, y) = 8x^3 + y^3 - 12xy + 6$ and classify them according to type (local maximum, local minimum, or saddle). (20 points)
6. Use the method of Lagrange multipliers to begin finding the extrema of the function $f(x, y, z) = x + 2y + 3z$ subject to the constraint that the points are on the unit sphere $x^2 + y^2 + z^2 = 1$. You do not have to complete the problem – just write down a system of equations which you would solve to find the critical points. (15 points)