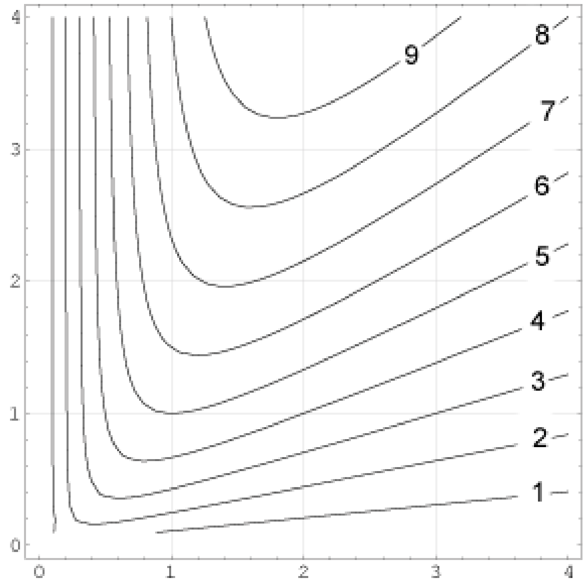
MA 234-01 §13.1–14.3	# Test #1 Solutions	**24 April 1997**

1. Suppose the level curves for a function $f(x, y)$ are as given. Use the level curve diagram to do the following.



   (a) Estimate the values of $f_x(1, 1)$ and $f_y(1, 1)$. *(10 points)*

   $f_x(1, 1) \approx \frac{2-3}{1} = -1$. $f_y(1, 1) \approx \frac{5-4}{1} = 2$.

   (b) Estimate the value of $f_u(2, 1)$ where u is a unit vector making an angle of $-\frac{Pi}{4}$ with the positive x-axis. *(10 points)*

   As you travel from (2,1) in the given direction for one unit, the function decreases about three units, i.e., $f_u(2, 1) \approx \frac{2-5}{1} = -3$.

   (c) Draw the gradient vector at the point (2,2), $\nabla f(2, 2)$ on the level curve diagram. Place the tail of the vector at (2,2). *(10 points)*

2. Let $f(x, y) = \sqrt{x^2 + y^4}$. Compute the differential of f and simplify the result. *(12 points)*

   $df(x, y) = f_x(x, y)dx + f_y(x, y)dy = \frac{x}{\sqrt{x^2+y^4}}dx + \frac{y^3}{\sqrt{x^2+y^4}}dy.$

3. A right circular cylinder is changing its radius and height with respect to time. Determine the rate of change of the volume of the cylinder when the radius is 3 inches and the height is 5 inches of the radius is increasing at 2 in/sec and the height is decreasing at 3 in/sec. *(11 points)*

   $\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt} = 2\pi rhr' + \pi r^2 h' = 2\pi(3)(5)(2) + \pi(3)^2(-3) = 33\pi \frac{in^3}{sec.}$

4. Find the second degree Taylor polynomial about the origin of the function $f(x, y) = \sin(xy)$. From this Taylor polynomial, conclude that the origin is a critical point of f and use the Taylor polynomial to determine which type of critical point you have found. *(12 points)*

   $f_x = y\cos(xy)$, $f_y = x\cos(xy)$, $f_{xx} = -y^2\sin(xy)$, $f_{yy} = -x^2\sin(xy)$, $f_{xy} = -xy\sin(xy) + \cos(xy)$. Evaluating all of these at the point $(0, 0)$ gives 0 except for $f_{xy}(0, 0) = 1$. Thus the Taylor polynomial of degree 2 about $(0, 0)$ is just xy, which is a hyperbolic paraboloid. Thus f has a saddle point at $(0, 0)$.

5. Find all critical points for the function $f(x, y) = 8x^3 + y^3 - 12xy + 6$ and classify them according to type (local maximum, local minimum, or saddle). *(20 points)*

   $f_x = 24x^2 - 12y = 0$ and $f_y = 3y^2 - 12x = 0$ gives two simultaneous solutions: $(0, 0)$ and $(2, 1)$. At $(0, 0)$, the discriminant D is negative saddle. At $(2, 1)$ the discriminant $D > 0$ and $f_x(2, 1) > 0$ local minimum.

6. Use the method of Lagrange multipliers to begin finding the extrema of the function $f(x, y, z) = x + 2y + 3z$ subject to the constraint that the points are on the unit sphere $x^2 + y^2 + z^2 = 1$. You do not have to complete the problem – just write down a system of equations which you would solve to find the critical points. *(15 points)*

   $\nabla f(x, y) = \lambda\nabla g(x, y)$ so we have $(1, 2, 3) = \lambda(2x, 2y, 2z)$ together with $x^2 + y^2 + z^2 = 1$.