| MA 234-01 §15.1–16.3 | **Test #2** | *score* | **Name:** ______________ **23 May 1997** |
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1. Evaluate $\iint_R y \sin^2 x \, dA$ where $R$ is the region in the first and second quadrants of the $xy$-plane below the curve $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$. *(16 points)*

2. Find the volume of the solid in 3-space which is below the surface $z = xy$ and above the region in the $xy$-plane bounded by the curves $y = \sqrt{x}$ and $y = x^2$. *(16 points)*

3. Find the moment of inertia about the $z$-axis of a sphere of radius 1 centered at the origin if the mass density at any point in the ball is given by $\rho(x, y, z) = \sqrt{x^2 + y^2}$. *(16 points)*

4. A square in the $xy$-plane with vertices $(2, 2)$, $(4, 0)$, $(6, 2)$, and $(4, 4)$ is revolved about the $y$-axis. Find the volume of the resulting solid of revolution. *(16 points)*

5. Find the volume of the solid in 3-space under the surface $z = 16 - x^2 - y^2$ and above the $xy$-plane. *(16 points)*

6. Let $\vec{r}(t) = (3\cos(2t), 3\sin(2t), 4t)$ for $0 \le t \le 2\pi$.
   (a) Compute the arclength of the curve from $t = 0$ to $t = \pi$. *(10 points)*
   (b) Compute the acceleration vector and show that it is always parallel to, but in the opposite direction as, the vector formed by the $x$- and $y$-components of the location vector $\vec{r}(t)$. *(10 points)*

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## Solutions

1. $\iint_R y \sin^2 \, dA = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\cos x} y \sin^2 x \, dy \, dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} y^2 \sin^2 x \Big|_{y=0}^{y=\cos x} dx = \frac{1}{2} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos^2 x \sin^2 x \, dx = \frac{1}{2}(2)\frac{\pi}{16} = \frac{\pi}{16}$

2. $V = \iint_R xy \, dV = \int_0^1 \int_{x^2}^{\sqrt{x}} xy \, dy \, dx = \int_0^1 \frac{1}{2} y^2 x \Big|_{y=x^2}^{y=\sqrt{x}} dx = \frac{1}{2} \int_0^1 (x^2 - x^5) \, dx = \frac{1}{2}(\frac{1}{3} - \frac{1}{6}) = \frac{1}{12}$

3. $I = \iiint (x^2 + y^2)\sqrt{x^2 + y^2} \, dV = \iiint (x^2 + y^2)^{\frac{3}{2}} \, dV = \int_0^{2\pi} \int_0^{\pi} \int_0^1 (\rho \sin\phi)^3 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = \int_0^{2\pi} \int_0^{\pi} \int_0^1 \rho^5 \sin^4 \phi \, d\rho \, d\phi \, d\theta = \frac{1}{6} 2\pi \frac{3 \cdot 1}{4 \cdot 2} \frac{\pi}{2} = \frac{\pi^2}{8}$

4. Since the square in the $xy$-plane has area $\left(2\sqrt{2}\right)^2$ and the centroid is $(4, 2)$, the volume of the solid is $V = 4 \cdot 2\pi \left(2\sqrt{2}\right)^2 = 64\pi$.

5. $V = \iint \left(16 - x^2 - y^2\right) \, dA = \int_0^{2\pi} \int_0^4 \left(16 - r^2\right) r \, dr \, d\theta = \int_0^{2\pi} \int_0^4 \left(16r - r^3\right) \, dr \, d\theta = 2\pi \left(8r^2 - \frac{r^4}{4}\right)\Big|_0^4 = 2\pi \cdot (32 - 16) = 2\pi(16 \cdot 8 - 4^3) = (128 - 64)2\pi = 128\pi$

6. (a) $L = \int_0^{\pi} \sqrt{(-6\sin(2t))^2 + (6\cos(2t))^2 + 4^2} \, dt = \int_0^{\pi} \sqrt{36 + 16} \, dt = \int_0^{\pi} \sqrt{52} \, dt = 2\sqrt{13}\,\pi$
   (b) $\vec{v}(t) = (-6\sin(2t), 6\cos(2t), 4)$ so $\vec{a}(t) = (-12\cos(2t), -12\sin(2t), 0)$. Since $\vec{a}(t) = -4 \cdot (3\cos(t) \cdot \vec{i} + 3\sin(t) \cdot \vec{j})$ the acceleration vector is parallel to but in the opposite direction as the first two components of the location vector.