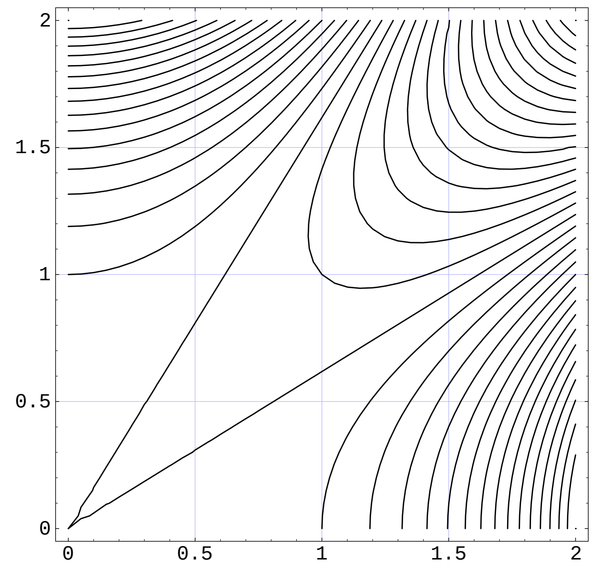
MA 234-01 §13.1–18.3	**Final Exam**	*score*	**Name:** ______________ **9 June 1997**

1. Suppose the level curves for a function $f(x, y)$ are as given. Use the level curve diagram to do the following.



   (a) Estimate the values of $f_x(1,1)$ and $f_y(1,1)$. *(4 points)*

   (b) Estimate the value of $f_u(1,1)$ where u is a unit vector making an angle of $\frac{3\pi}{4}$ with the positive x-axis. *(4 points)*

   (c) Draw the gradient vector at the point (1,1), $\nabla f(1,1)$ on the level curve diagram. Place the tail of the vector at (1,1). *(5 points)*

2. Let $f(x, y) = \sqrt{x^2 + y^2}$. Compute the differential of f and simplify the result. Use the differential of f to approximate $\sqrt{2.98^2 + 3.01^2}$. *(6 points)*

3. A right circular cone ($V = \frac{1}{3}\pi r^2 h$) is changing its base radius and height with respect to time. Determine the rate of change of the volume of the cylinder when the base radius is 2 inches and the height is 6 inches of the radius is increasing at 2 in/sec and the height is decreasing at 1 in/sec. *(6 points)*

4. Find the second degree Taylor polynomial about the origin of the function $f(x, y) = \ln(x^2 + y^2 + 1)$. From this Taylor polynomial, conclude that the origin is a critical point of f and use the Taylor polynomial to determine which type of critical point you have found. *(6 points)*

5. Find all critical points for the function $f(x, y) = 4xy - x^4 - y^4$ and classify them according to type (local maximum, local minimum, or saddle). *(10 points)*

6. Use the method of Lagrange multipliers to begin finding the extrema of the function $f(x, y, z) = x^2 - y^2 - z$ subject to the constraint that the points are on the unit ellipsoid $x^2 + y^2 + 4z^2 = 4$. You do not have to complete the problem – just write down a system of equations which you would solve to find the critical points. *(6 points)*

7. Evaluate $\displaystyle\iint_R y \cos x \, dy \, dx$ where R is the region in the first quadrant of the xy-plane below the curve $y = \sin x$ between $x = 0$ and $x = \frac{\pi}{2}$. *(6 points)*

8. Find the volume of the solid in 3-space which is below the surface $z = x^2 + y^2$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$ and $y = x^2$. *(6 points)*

9. Use spherical coordinates to evaluate the integral

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} xz^2\, dz\, dy\, dx.$$

(7 points)

10. A circular disk in the xy-plane with center $(3,1)$ and radius 2 is revolved about the y-axis. Find the volume of the resulting solid of revolution. *(6 points)*

11. Find the volume of the solid in 3-space under the surface $z = 25 - x^2 - y^2$ and above the xy-plane. *(6 points)*

12. Compute the arclength of the curve given by $r(t) = (4\cos(3t), 4\sin(3t), 2t)$ for $0 \le t \le 2\pi$. *(6 points)*

13. Find two different parameterizations of the surface of revolution obtained by rotating the curve given by $z = x^2 + 1$ in the xz-plane about the z-axis. *(6 points)*

14. Let C be the curve in the xy-plane which begins at (0,2) and ends at (2,0) and travels along the quarter circle of radius 2 centered at the origin in a clockwise direction. In this problem we evaluate the line integral $\int_C (x^2\vec{i} + y^3\vec{j}) \cdot \vec{dr}$ in two ways.

    (a) Evaluate by parameterizing the curve. *(5 points)*

    (b) Evaluate by expressing the integrand as a gradient field and using the fundamental theorem of line integrals. *(5 points)*