MA 234-51 **Test #1** **Name:** ______________________

§13.1 – §14.7 **2 February 1993**

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1. Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

   (a) Compute $\mathrm{comp}_{\mathbf{a}}\mathbf{b}$ and $\mathrm{proj}_{\mathbf{a}}\mathbf{b}$. *(8 points)*

   (b) Find all vectors of magnitude 3 which are orthogonal to both $\mathbf{a}$ and $\mathbf{b}$. *(8 points)*

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2. The two planes given by $2x + y - z = 10$ and $x - y + 2z = 5$ intersect in a line. Find a set of symmetric equations for the line of intersection. *(9 points)*

3. Determine if the two lines given by $\frac{x-1}{1} = \frac{y+1}{1} = \frac{z-2}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z-1}{2}$ are skew. *(9 points)*

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4. Find the exact distance from the point $(2, 1, -1)$ to the line given by $x = 2 + t$, $y = 2 + t$, and $z = 2 - t$. *(9 points)*

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5. Find an equation of the plane which contains the point $(2, 3, -1)$ and the line given by $\frac{x}{2} = \frac{y+1}{1} = \frac{z-1}{-1}$. *(10 points)*

6. Let $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{2}{3}t^3\mathbf{k}$. Calculate the unit tangent vector function $\mathbf{T}(t)$. Also, find all points along the curve given by $\mathbf{r}(t)$ where $\mathbf{T}(t)$ is orthogonal to $\mathbf{r}''(t)$. *(10 points)*

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7. Let $\mathbf{r}(t)$ denote the vector location of a particle at time t. Suppose that $\mathbf{r}'(0) = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{r}''(0) = \mathbf{i} - \mathbf{j} - 3\mathbf{k}$.

   (a) Compute the tangential and normal components of acceleration when $t = 0$. *(9 points)*

   (b) Compute the value of the curvature when $t = 0$. *(9 points)*

8. Let $r(t) = \cos \pi t \, \mathbf{i} + \sin \pi t \, \mathbf{j} + 2t \, \mathbf{k}$. This space curve contains the points $(1, 0, 0)$ and $(-1, 0, 6)$. Find the length of the curve between those two points. *(9 points)*

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9. Consider the plane curve given by the vector equation

$$\mathbf{r}(t) = (\cos t + t \sin t) \, \mathbf{i} + (\sin t - t \cos t) \, \mathbf{j}.$$

Find a formula for the curvature at each point of the curve. *(10 points)*