**MA 234-51** **Test #2** **Name:** ____________

**§15.1 – §17.4** **4 March 1993**

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1. Let $f(x,y) = \sqrt{16 - 4x^2 + y^2}$.
   (a) Determine the domain of $f(x,y)$ and sketch the graph of the domain in the $xy$-plane using the left set of axes. *(4 points)*
   (b) Graph (on the right set of axes) the level curves corresponding to $z = 0$, $z = 4$, and $z = 5$. Label each curve. *(4 points)*

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2. For each of the following equations, determine if there is a closely matching graph on the last page. If there is, report its number. If there is not, write NO MATCH. *(12 points)*
   (a) $x^2 + 1 = y^2 + z^2$ . . . . . . . . . . . . . . . . . . . . . ____________
   (b) $z^2 = x^2 + y^2 + 1$ . . . . . . . . . . . . . . . . . . . . . ____________
   (c) $z = \sqrt{x^2 + y^2}$ . . . . . . . . . . . . . . . . . . . . . ____________
   (d) $z = x^2 - y^2$ . . . . . . . . . . . . . . . . . . . . . ____________
   (e) $z = x^2 + 16y^2$ . . . . . . . . . . . . . . . . . . . . . ____________
   (f) $x^2 + 4y^2 + 25z^2 = 100$ . . . . . . . . . . . . . . . . . . ____________

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3. Answer *true* if the statement is always true; otherwise answer *false*: *(3 points each)*
   (a) If $f(x,y)$ is differentiable at a point, the $f(x,y)$ must be continuous at that point.
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   (b) If $f(x,y)$ has continuous first partial derivatives everywhere, then $f(x,y)$ must be differentiable everywhere. ____________

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4. Let $f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$. Determine if $f(x,y)$ is continuous at $(0,0)$.
   *(10 points)*

5. Let $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2}}$.

   (a) Calculate the gradient of $f$, $\nabla f$. *(6 points)*

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   (b) Find the rate of change of $f$ at the point (3,4) in the direction straight into the origin. *(6 points)*

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   (c) If a small bug is on the graph of $f$ over the point $(3,4)$ in the $xy$-plane, in which direction should the bug begin to walk along the surface so that his/her $z$-level is unchanged? Express you answer as a vector in the $xy$-plane. *(6 points)*

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6. A right circular cone is changing shape so that its radius in constantly increasing at a rate of 2 inches per second while its height is constantly decreasing at a rate of 1 inch per second. Find the rate of change of the volume of the cone when the radius has value 5 inches and the height has value 15 inches. *(8 points)*

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7. Let $f(x,y) = 3x^2 - y^4$. Find an equation of the tangent plane to the graph of $f$ when $x = 2$ and $y = 1$. *(10 points)*

8. Find all the critical points for the function $f(x,y) = 3x^2y + y^3 - 108y + 1$ and classify them according to type. *(12 points)*

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9. Compute the value of $\displaystyle\iint_R (x^2 - xy)\,dA$ where $R$ is the region in the $xy$-plane bounded by the two curves $y = x^2$ and $x = y^2$. *(9 points)*

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10. Compute the value of $\displaystyle\int_0^1 \int_y^1 e^{x^2}\,dx\,dy$. *(9 points)*