MA 237-02 §1.1 – 2.3	**Test #1**	*score*	**Name:** ____________ **5 October 2001**

INSTRUCTIONS: Work the following problems on your own paper.

1. Consider the following system of equations.

$$\begin{aligned} 2x_1 - 3x_2 + 2x_3 &= 2 \\ 2x_1 - 2x_2 + x_3 &= 4 \end{aligned}$$

   Write out the augmented matrix for the system of equations. Then use row operations to transform the augmented matrix into reduced echelon form. Explain each row operation you use, e.g., with notation like $R_2 \to R_2 - R_1$. Then write the solutions in parametric form and describe the set of solutions geometrically. *(20 points)*

2. Consider the following system of equations.

$$\begin{aligned} 2x_1 - x_2 + 2x_3 + 6x_4 + 2x_5 &= 2 \\ 2x_1 - x_2 + x_3 + 4x_4 + x_5 &= 4 \\ x_1 - x_2 + 2x_3 + 5x_4 + x_5 &= 1 \\ 2x_1 - x_2 + x_3 + 5x_4 + 2x_5 &= 5 \end{aligned}$$

   Suppose that the augmented matrix for this system has echelon form

$$\begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 0 & -1 & -4 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   (a) Express the set of solutions in parametric form (i.e., as a sum of a translation vector plus linear combinations of other vectors). Describe the set of solutions geometrically. *(10 points)*
   (b) Find a basis for the null space and state its dimension. *(10 points)*
   (c) Find a basis for the row space and state its dimension. *(10 points)*
   (d) Find a basis for the column space and state its dimension. *(10 points)*

3. For each of the following, write TRUE if the statement is always true. Otherwise, write FALSE. Give a brief explanation. *(5 points each)*

   (a) Any two vectors in $\mathbb{R}^3$ are independent.
   (b) Any four vectors in $\mathbb{R}^3$ are dependent.
   (c) For any matrix A, the dimension of the row space equals the dimension of th ecolumn space.
   (d) For any $m \times n$ matrix A, the rank of A equals n minus the dimension of the nullspace of A.
   (e) A matrix equation of the form $AX = B$ has no solutions if and only if B is not in the column space of A.
   (f) The dimension of the vector space of $m \times n$ matrices, $M(m, n)$, is $m + n$.

4. The accompanying diagram shows the traffic flow (in cars per minute) on a collection of one-way streets. Find a system of equations that gives the relationships among the variables. You do not need to solve the system. *(10 points)*