MA 237-02 §3.1 – 6.1	**Test #2**	*score*	**Name:** ______________ **19 November 2001**

INSTRUCTIONS: Answers to question 7 may be written on this page. All other problems should be worked on a separate sheet.

1. Find a matrix that induces a transformation from $\mathbb{R}^2$ to itself that sends the standard unit square in the first quadrant (with vertices (0,0), (1,0), (1,1), and (0,1)) to the parallelogram with vertices (0,0), (2,-1), (1,-2), and (-1,-1). How many such matrices are possible? *(10 points)*

2. Give an example of two matrices A and B such that $AB \neq BA$, if such an example exists. *(10 points)*

3. Use the augmented matrix method to find (by hand) the inverse of the following matrix. *(10 points)*

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

4. Two $n \times n$ matrices A and B are called *similar* if there exists an invertible matrix Q such that $A = QBQ^{-1}$. Prove that similar matrices always have the same determinant. *(10 points)*

5. Find the projection of the vector $[1,2,3]^t \in \mathbb{R}^3$ onto the subspace of $\mathbb{R}^3$ spanned by the two vectors $[2,1,2]^t$ and $[1,2,1]^t$. *(10 points)*

6. Show that the vector X is an eigenvector for the matrix A and determine the corresponding eigenvalue. *(10 points)*

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} 5 \\ 8 \\ 7 \end{bmatrix}$$

7. For each of the following, answer TRUE if the given statement in always true. Otherwise, answer FALSE. *(5 points each)*

   (a) For the vector $[1,2,3]^t \in \mathbb{R}^3$, its coordinates in the basis $[2,1,0]^t$, $[1,0,4]^t$, $[1,-1,0]^t$ are $[1,1,-2]^t$. ____________

   (b) No linear transformations from $\mathbb{R}^4$ to $\mathbb{R}^3$ are one-to-one. ____________

   (c) All linear transformations from $\mathbb{R}^3$ to $\mathbb{R}^4$ are one-to-one. ____________

   (d) A linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$ is one-to-one if and only if it is onto. ____________

   (e) If A' is obtained from a square matrix A by replacing all of the entries of A by their negatives, then $\det(A') = -\det(A)$. ____________

   (f) For an invertible matrix A, $\det(A^{-1}) = \dfrac{1}{\det(A)}$. ____________

8. Short Answer *(5 points each)*

   (a) Suppose A is a 5×5 matrix and that the dimension of the nullspace of A is 2. Find the dimension of the image of the transformation. Briefly explain.

   (b) Let A be a 3×4 matrix and B a 4×3 matrix. For the transformations determined by the matrix products AB and BA, describe whether or not either can be one-to-one or onto.