MA 237-01 §1.1 – 2.3	# Test #1	*score*	**Name:** ____________ **26 February 2002**

INSTRUCTIONS: Work the following problems on your own paper.

1. Consider the following system of equations.

$$\begin{aligned} 2x_1 - x_2 + x_3 &= 1 \\ 3x_1 - 2x_2 - x_3 &= 2 \end{aligned}$$

Write out the augmented matrix for the system of equations. Then use row operations to transform the augmented matrix into reduced echelon form. Document each row operation you use, e.g., with notation like $R_2 \to R_2 - R_1$. Then write the solutions in parametric form and describe the set of solutions geometrically. *(16 points)*

2. For the following system of equations

$$\begin{aligned} 3x_1 - 2x_2 - 2x_3 + x_4 + 13x_5 &= 15 \\ 3x_1 + 2x_2 - 10x_3 + 2x_4 + 2x_5 &= 0 \\ x_1 + 2x_2 - 6x_3 - 2x_4 - 6x_5 &= -8 \end{aligned}$$

suppose that the augmented matrix for this system has echelon form

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 2 & 2 \\ 0 & 1 & -2 & 0 & -3 & -4 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(a) Express the set of solutions in parametric form (i.e., as a sum of a translation vector plus linear combinations of other vectors). Describe the set of solutions geometrically. *(8 points)*

(b) Regarding the original system of equations as a matrix equation in the form $AX = B$, find a basis for the null space of A and state its dimension. *(8 points)*

(c) Find a basis for the row space of the coefficient matrix A and state its dimension. *(8 points)*

(d) Find a basis for the column space of A and state its dimension. *(8 points)*

3. For each of the following, write TRUE if the statement is always true. Otherwise, write FALSE. Give a brief explanation. *(4 points each)*

(a) It is possible for a system of linear equations with more unknowns that equations to have no solutions.

(b) The set of solutions of any system of linear equations in n unknowns forms a subspace of $\mathbb{R}^n$.

(c) Any two vectors in $\mathbb{R}^4$ are independent.

(d) If A is a 5×3 matrix, the rank of A is less than or equal to 3 and the dimension of the nullspace is 2 or larger.

(e) Any 4 vectors in $\mathbb{R}^3$ are dependent.

(f) For any matrix A, the dimension of the row space equals the dimension of the column space.

(g) For any $m \times n$ matrix A, the rank of A equals n minus the dimension of the nullspace of A.

(h) The dimension of the vector space of $m \times n$ matrices, $M(m, n)$, is $m \cdot n$.

(i) A homogeneous system of linear equations cannot be inconsistent.

4. Construct a 3×4 matrix A so that the equation $AX = B$ has a solution if and only if $B \in \text{span}([2, 1, 3]^t)$. *(8 points)*

5. The accompanying diagram shows the traffic flow (in cars per minute) on a collection of one-way streets. Find a system of equations that gives the relationships among the variables. You do not need to solve the system. *(8 points)*