MA 237-02 §3.1 – 6.1	**Test #2**	*score*	**Name:** ____________ **18 April 2002**

INSTRUCTIONS: Answers to question 8 may be written on this page. All other problems should be worked on a separate sheet.

1. Find the coordinates of the point $[-1,2]^t$ in the basis $\{[2,1]^t,[1,1]^t\}$ for $\mathbb{R}^2$. Show how you do this. *(10 points)*

2. Give an example of a 2×3 matrix A so that the image of the induced transformation of A consists of the line $y = 2x$ in the plane. State the domain, range, dimension of the image, and dimensionn of the null space for such a transformation? Briefly explain. *(10 points)*

3. Give an example of two matrices A and B such that $(AB)^t \neq A^tB^t$ (show this), or state that such an example can't occur. *(10 points)*

4. Use the augmented matrix method to find (by hand) the inverse of the following matrix. *(10 points)*

$$\begin{bmatrix} -1 & 3 & 1 \\ -1 & 2 & 1 \\ -1 & 0 & 2 \end{bmatrix}$$

5. Use the Gram-Schmidt process to convert the ordered basis $\{[1,1,1]^t,[2,1,2]^t,[1,-1,-1]^t\}$ into an orthogonal basis. Show your work. *(10 points)*

6. Calculate the characteristic polynomial for the given matrix and determine all of the eigenvalues. Show your work. You do not need to find any eigenvectors on this problem. *(10 points)*

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

7. Since the matrix below is in triangular form, you know that $\lambda = 2$ is an eigenvalue for the matrix. Determine the corresponding eigenspace by exhibiting an eigenvector (or collection of independent eigenvectors) that span(s) the eigenspace. Show your work. *(10 points)*

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

8. For each of the following, answer TRUE if the given statement in always true. Otherwise, answer FALSE. *(3 points each)*

   (a) Any subspace of $\mathbb{R}^n$ has an orthogonal basis. ____________

   (b) For any invertible matrix A, $|A^{-1}| = \dfrac{1}{|A|}$. ____________

   (c) For any square matrices A and B of the same size, $|AB| = |BA|$. ____________

   (d) If a matrix B is obtained from an $n\times n$ matrix A by interchanging exactly two rows, then $|A| = |B|$. ____________

(e) Any linear transformation from $\mathbb{R}^1$ to $\mathbb{R}^2$ is one-to-one. ____________

(f) A square matrix with two identical columns has a determinant of 0. ____________

(g) A square matrix is invertible if and only if the associated linear transformation is onto. ____________

(h) If A is a square matrix, and if $AX = B$ has no solutions for some vector B, then A is not invertible. ____________

(i) If A is a 5×3 matrix and B is a 3×4, the transformation induced by the product matrix AB is never one-to-one. ____________

(j) If A is a 5×3 matrix and B is a 3×4, the transformation induced by the product matrix AB is never onto. ____________