| MA 238-01<br>§1.1–1.7,1.9<br>§2.1,2.5 | **Test #1** | *score* | **Name:** ____________<br>**6 October 1999** |
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1. For each of the following differential equations, state whether or not it is linear (L), separable (S), homogeneous (H), or none (N). State all that apply. *(9 points)*

   (a) $\dfrac{dy}{dt} = ty^2 \sin(t)$ . . . . . . . . . . . . . . . . . . ____________

   (b) $t^2 \dfrac{dy}{dt} + \ln(t) \cdot y = t^3 + 3t - 7$ . . . . . . . . . . . . ____________

   (c) $(x^2 + xy)dy = (y^2 - x^2)dx$ . . . . . . . . . . . . ____________

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2. Find the general solution to the differential equation $\dfrac{dy}{dt} + \dfrac{2}{t}y = t$. Then find the solution that satisfies the initial condition $y(1) = 2$. What happens to $y$ as $t \to \infty$? As $t \to 0$? *(15 points)*

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3. Find the solution for the initial value problem $\dfrac{dy}{dt} = 2y^2 + ty^2$. At what $t$-value do all of the solutions have their minimum value? Explain. *(15 points)*

4. A 100-liter tank contains pure water. A brine solution containing $250\sin(t)$ grams of salt per liter at time $t$ minutes flows into the tank at a rate of 2 liters per minute. Write an initial value problem the solution to which would give the amount of salt in the tank at time $t$. You do not need to solve the IVP. *(15 points)*

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5. Bugs are located in the $(x, y)$-plane at the four points $(\pm 1, \pm 1)$. The bug in quadrant I pursues the bug in quadrant IV; the bug in quadrant IV pursues the bug in quadrant III; etc. Find an IVP the solution to which would describe the path the bug in quadrant 1 follows. You do not need to solve the IVP. *(15 points)*

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6. Use the method of Picard to find approximations to the solution of the differential equation $y' = y + \sin(t)$, $y(0) = 0$. You should compute the Picard iterates $y_0$, $y_1$, and $y_2$. *(15 points)*

7. For the differential equation $\frac{dy}{dt} = 2t - y$ with initial condition $y(0) = 1$, use Euler's method to approximate the value of $y(1)$ using 2 steps (i.e., use $h = 0.5$). Then use the Runge-Kutta method to estimate $y(1)$ using just one step (i.e., use $h = 1$). Which estimate do you think is the more accurate? Why? *(16 points)*

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EULER METHOD

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RUNGE-KUTTA METHOD