| $\begin{aligned} & \text { MA 238-02 } \\ & \S 3.1-3.7,4.1 \\ & \S 4.2,4.4,6.1 \end{aligned}$ | Test $\# 2$ | score | Name: $\quad 2$ November 1999 |
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1. Find the general solution of $y^{\prime \prime}+2 y^{\prime}+5 y=e^{-2 t}$ by first solving the corresponding undriven equation, then using the method of undetermined coefficients. (12 points)
2. Is it possible for $\left\{e^{t}, \sin (t)\right\}$ to form a basic set of solutions on the interval $[0,4]$ for a differential equation of the form $y^{\prime \prime}+a(t) y^{\prime}+b(t) y=0$ where $a(t)$ and $b(t)$ are both continuous on $[0,4]$ ? Explain. (7 points)
3. A rod of negligible mass is suspended from a high ceiling on a pivot. Your job is to estimate the length of the rod, but the only tools you have available are a 1 kg mass and a stopwatch. So you attach the mass to the end of the rod and find that, with a small displacement, the resulting pendulum completes 8 swings in 20 seconds. Estimate the length of the rod and explain how you arrive at your answer. (10 points)
4. Consider the equation $y^{\prime \prime}+y=\sec t\left(-\frac{\pi}{2}<t<\frac{\pi}{2}\right)$. Begin the variation of parameters method to find the general solution, but you may stop once you have computed $u_{i}^{\prime}(t)$ and $u_{2}^{\prime}(t)$ in the interest of saving time. (12 points)
5. Use the definition of the Laplace transform to find the Laplace transform of $e^{2 t}$. Be sure to evaluate the improper integral carefully so your work can be followed. State the $s$-interval over which the transform exists. (10 points)
6. Find the Laplace transform of the solution of the IVP $y^{\prime \prime}+y^{\prime}+2 y=\sin (t), y(0)=1$, $y^{\prime}(0)=-1 . \quad(10$ points $)$
7. The orbit diagram for a pendulum is given below. Describe the motion of the pendulum and the IVP that produced the orbit diagram (linear or non-linear, damped or undamped, estimate the initial conditions, tumbles over top of pivot or not - if so how many times, etc.). (10 points)

8. The graph of $\theta$ versus $t$ for a pendulum is given below. Describe the motion using ideas from the previous problem. Then sketch the orbit diagram for this motion (12 points)


