MA 238-01 Ch. 1-4, 6	**Final Exam**	*score*	**Name:** ______ **1 August 2001**

NOTE: Be sure to show all of your work in order to receive credit for a correct answer.

1. Find all solutions for the given differential equation and then find the solution that satisfies the initial condition $y(0) = 0$. *(9 points)*

$$\frac{dy}{dt} + 2ty = t$$

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2. Find the solution (an implicit solution is acceptable) for the initial value problem *(8 points)*

$$\frac{dy}{dt} = \frac{y\cos t}{1 + 2y^2}\ , \qquad y(0) = 1$$

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3. Let $y(t)$ denote the size of a population, say in thousands, of creatures, and suppose that y satisfies the differential equation

$$\frac{dy}{dt} = -y^2 + 10y - 16$$

Determine if there are any equilibrium solutions. Discuss the features of this system (use words like *harvesting*, *stable*, *carrying capacity*, etc.). Sketch a representative collection of solution curves on the axes provided. *(8 points)*

4. Use the method of Picard to find approximations to the solution of the differential equation $y' = t^2 y - t,\ \ y(0) = 0$ by computing the Picard iterates y_0, y_1, and y_2. *(9 points)*

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5. For the differential equation $\dfrac{dy}{dt} = y - t^2$ with initial condition $y(0) = 2$, use Euler's method to approximate the value of $y(1)$ using 2 steps (i.e., use $h = 0.5$). Then use the Runge-Kutta method to estimate $y(1)$ using just one step (i.e., use $h = 1$). *(10 points)*

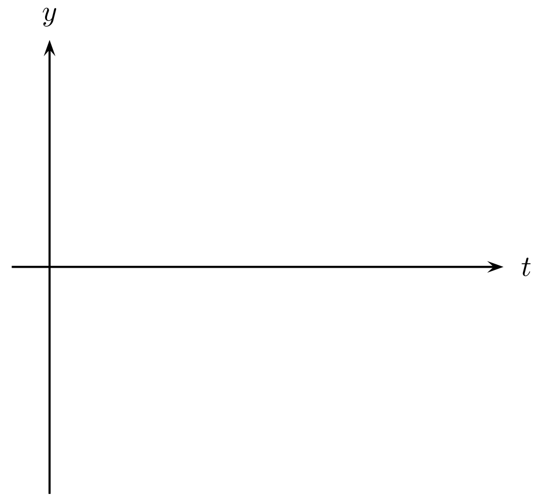
EULER METHOD

RUNGE-KUTTA METHOD

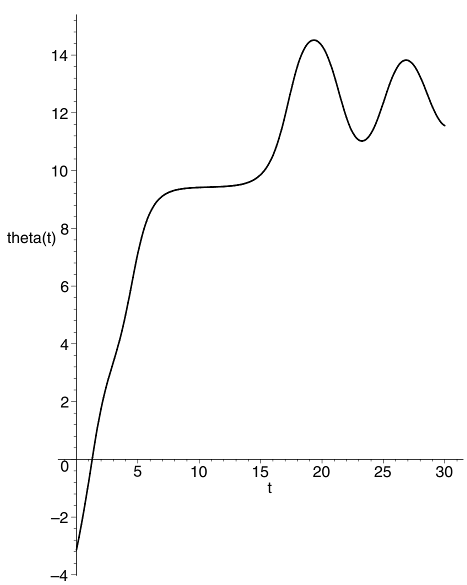
6. A tank contains 100 gallons of contaminated water in which 10 pounds of pollutants are dissolved. Contaminated water starts to run into the tank at a rate of 5 gallons per minute. The concentration of pollutant in the incoming water is 2 pounds per gallon. The solution in the tank is thoroughly mixed, and contaminated water flows out a the rate of 5 gallons per minute. Write down the IVP for this problem, but do not solve it. *(8 points)*

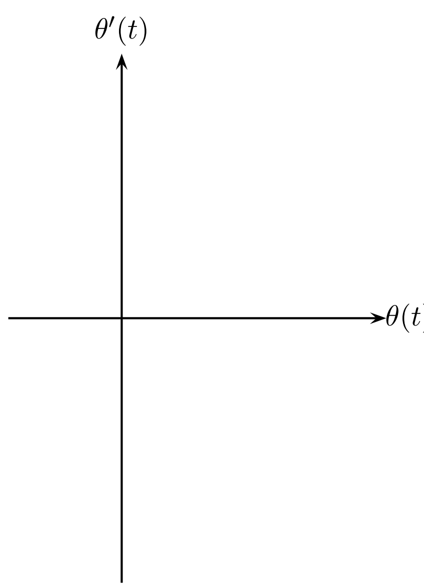
7. Find the general solution of $y'' - 3y' - 4y = \cos(2t)$ by first solving the corresponding undriven equation, then using the method of undetermined coefficients. *(11 points)*

8. A long spring that is suspended from a ceiling. If a 2 kilogram mass is attached to the spring, it stretches the spring 1 meter (i.e., the new equilibrium position is 1 meter lower). If the mass is pulled down an additional 2 meters and released from rest, find the equation of motion for the mass (assuming no damping). What damping should be used in this system in order to produce a critically damped system? Sketch the graphs of the undamped and critically damped solutions on the axes provided. *(9 points)*



9. A pendulum is suspended from a pivot as discussed in class. The variable θ measures the angle (in radians) of the pendulum arm from the downward vertical position. The graph shows a plot of θ versus t for a damped, true (non-linearized) pendulum. Estimate the initial conditions. Describe the motion of the pendulum based on what you see in the graph. Estimate the value of t when the pendulum bob first passes over the top (its highest position) and also when the bob first passes through its lowest position. On the empty set of axes, sketch the phase plane graph of $\theta'(t)$ versus $\theta(t)$ for the graph on the left. *(10 points)*





10. Find the functions $f(t)$ whose Laplace transforms are as follows: *(5 points each)*

    (a) $\mathcal{L}[f](s) = \dfrac{1}{s^2 - 1}$

    (b) $\mathcal{L}[f](s) = \dfrac{se^{-3s}}{s^2 + 9}$

11. Find the Laplace transform of the solution of the IVP

$$2y'' + y' + 2y = h(t) \qquad y(0) = 0 \quad y'(0) = 0$$

where $h(t) = u_5(t) - u_{20}(t) = \text{step}(t-5) - \text{step}(t-20)$. Note: You do not need to solve this IVP, you just need to find the Laplace transform. *(9 points)*

Note: You do not need to solve this IVP, you just need to find the Laplace transform. *(8 points)*