

1. A 1000-liter tank is full of a brine solution containing 100 kg of salt initially. Salt water is flowing into the tank at a rate of 5 liters per minute at a concentration of 0.5 kg salt per liter and liquid is leaving the tank at the same rate. Find a formula for the amount of salt in the tank at time t , and explain what happens as $t \rightarrow \infty$. (7 points)

$$y' = \frac{5}{2} - \frac{5y}{1000} \quad y(0) = 100$$

$$y = 500 - 400 e^{-t/200}$$

2. Ten milligrams of radioactive material with a half-life of one hour are injected into a patient so that a scan can be done. If the patient must have at least 2 mg of the radioactive material in his/her body during the scan, by how long past the injection time must the scan be completed? Express your answer in hours and minutes (rounded to the nearest minute). (6 points)

$$y(0) = 10$$

$$y' = -ky$$

$$y = C e^{-kt} = 10 e^{-kt}$$

$$5 = 10 e^{-k}$$

$$e^k = 2$$

$$k = \ln 2$$

$$y = 10 \cdot 2^{-t}$$

$$2 = 10 \cdot 2^{-t}$$

$$2^t = 5$$

$$t = \log_2 5 = \frac{\ln 5}{\ln 2} \approx 2.32193 \text{ hr}$$

$$\approx 2 \text{ hours } 19 \text{ minutes}$$

3. A wiffle ball of mass m is thrown upward with an initial velocity of 30 meters per second. If air resistance is neglected, how long will the ball remain aloft? If $k/m = 0.5/\text{second}$ where k is the viscous damping constant, approximate how long will the ball remain aloft. (7 points)

w/o resistance: $y = -4.9t^2 + 30t = t(30 - 4.9t) \Rightarrow t_f = \frac{30}{4.9} \approx 6.12 \text{ seconds}$

w/ resistance: $y' = v$
 $v' = -9.8 - \frac{1}{2}v$

$$t_f \approx 4.54 \text{ seconds}$$