MA 238-02 **Ch. 1-4, 6**	**Final Exam**	*score*	**Name:** ______________________ **2 May 1999**

1. Find all solutions for the differential equation *(9 points)*

$$2\frac{dy}{dt} - y = 2e^{-t}$$

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2. Find the solution (an implicit solution is acceptable) for the initial value problem *(8 points)*

$$\frac{dy}{dt} = \frac{t^2}{1 - y^2} \qquad y(0) = 1$$

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3. A tank contains 100 gallons of contaminated water in which 10 pounds of pollutants are dissolved. Contaminated water starts to run into the tank at a rate of 5 gallons per minute. The concentration of pollutant in the incoming water is 2 pounds per gallon. The solution in the tank is thoroughly mixed, and contaminated water flows out a the rate of 5 gallons per minute. Write down the IVP for this problem, but do not solve it. *(8 points)*

4. Use the method of Picard (Emile, not Jean-Luc) to find approximations to the solution of the differential equation $y' = ty + 1$, $y(0) = 0$ by computing the Picard iterates y_0, y_1, and y_2. *(9 points)*

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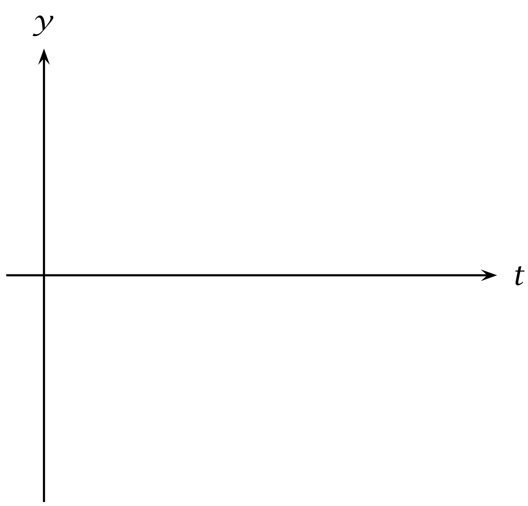
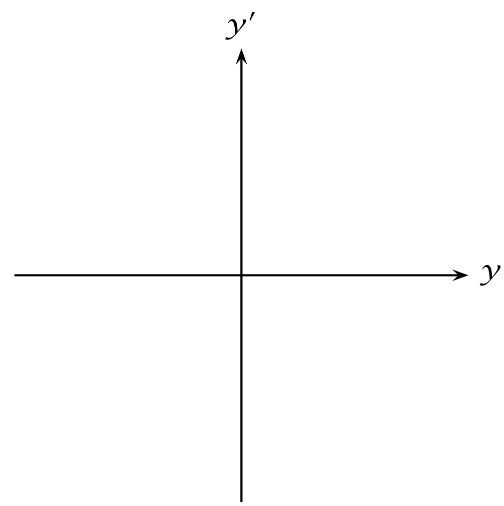
5. For the differential equation $\frac{dy}{dx} = x - y$ with initial condition $y(0) = 1$, use Euler's method to approximate the value of $y(1)$ using 2 steps (i.e., use $h = 0.5$). Then use the Runge-Kutta method to estimate $y(1)$ using just one step (i.e., use $h = 1$). *(10 points)*

EULER METHOD

RUNGE-KUTTA METHOD

6. A mass is attached to a spring that is suspended from the ceiling. If the spring is stretched 1 meter by pulling the mass down and if the mass is released from rest at time $t = 0$, sketch a reasonable graph of the displacement $y(t)$ of the mass from equilibrium at time t. Assume that there is a very small amount of damping so that the spring is underdamped. Also, sketch a reasonable graph of the phase plane plot of velocity versus displacement covering at least three"cycles" of the mass. *(9 points)*





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7. Find the general solution of the differential equation $y'' - y' - 6y = 5\cos(t)$. *(9 points)*

8. For the differential equation

$$t^2 y'' - 2y = 3t^2 - 1 \qquad t > 0$$

verify that $y_1(t) = t^2$ and $y_2(t) = t^{-1}$ form a basic set of solutions for the corresponding undriven differential equation. Then use the method of variation of parameters to find a particular solution to the driven equation. *(9 points)*

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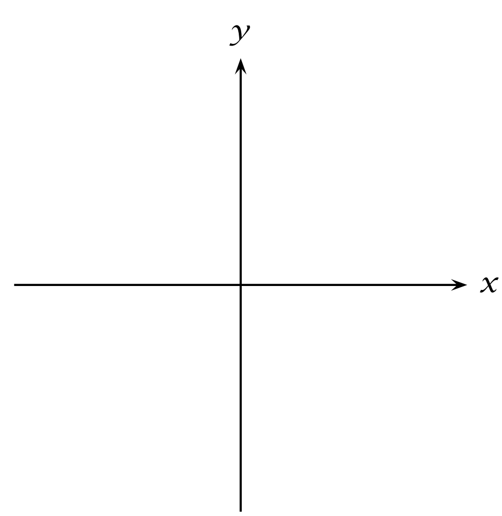
9. A pendulum is suspended from a pivot as discussed in class. The variable θ measures the angle (in radians) of the pendulum arm from the downward vertical position. The graph shows a plot of θ versus t for a damped (non-linearized) pendulum. Discuss the motion of the pendulum based on what you see in the graph. Estimate the value of t when the pendulum bob first passes over the top (its highest position) and also when the bob first passes through its lowest position. On the empty set of axes, sketch the phase plane graph of $\theta'(t)$ versus $\theta(t)$. *(10 points)*



10. Find the functions $f(t)$ whose Laplace transforms are as follows: *(5 points each)*

    (a) $\mathcal{L}[f](s) = \dfrac{1}{s^2 - 1}$

    (b) $\mathcal{L}[f](s) = \dfrac{se^{-3s}}{s^2 + 9}$

11. Find the Laplace transform of the solution of the IVP

$$2y'' + y' + 2y = h(t) \qquad y(0) = 0 \quad y'(0) = 0$$

where $h(t) = u_5(t) - u_{20}(t) = \text{step}(t-5) - \text{step}(t-20)$. Note: You do not need to solve this IVP, you just need to find the Laplace transform. *(9 points)*