MA 238-02 §1.1–1.8,2.1,2.5	**Test #1**	*score*	**Name:** ______________ **1 March 1999**

1. First find the general solution to the differential equation $\frac{dy}{dt} + 2ty = t$. Then find the solution that satisfies the initial condition $y(0) = 0$. What happens to y as $t \to \infty$? *(14 points)*

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2. Find the solution for the initial value problem $\frac{dy}{dt} = 2y^2 + ty^2$. At what t-value do all of the solutions have their minimum value? *(14 points)*

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3. Find all equilibrium solutions for the differential equation $y' = -y^2 + 12y - 20$. Sketch a graph of the vector field in the first quadrant of the ty-plane (you don't need to be very accurate – just get the vectors going up or down correctly according to the sign of y'). If this DE described the size of a population over time, discuss the characteristics of the population, e.g., is there a carrying capacity?, does the population grow without bound?, is there harvesting?, etc. *(14 points)*

4. In a certain culture of bacteria, the number of bacteria increased sixfold in 10 hours. How long did it take for the populatin to double? What assumptions are you making? *(14 points)*

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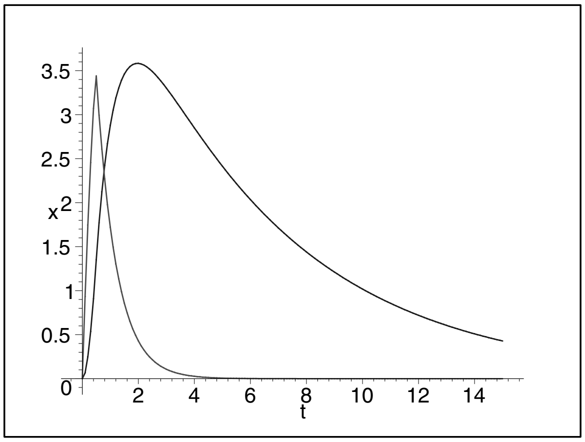
5. A single dose of medication is taken orally. The amount of medication in the intestines and in the blood is modelled with a 2-compartment model as was done in class. The graphs of $x(t) =$ "the amount of medicine in the intestines" and $y(t) =$ "the amount of medicine in the blood" are given below. By estimating the half-lives, write down a system of differential equations that could model this single dose. Next, if the dose is repeated each 4 hours, estimate (and explain your estimate) the amount of medicine that will be in the blood in the long run. *(14 points)*



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6. Use the method of Picard to find approximations to the solution of the differential equation $y' = -\frac{y}{2} + t,\ \ y(0) = 0$. You should compute the Picard iterates y_0, y_1, and y_2. *(14 points)*

7. For the differential equation $\frac{dy}{dx} = 2y - x$ with initial condition $y(0) = 1$, use Euler's method to approximate the value of $y(1)$ using 2 steps (i.e., use $h = 0.5$). Then use the Runge-Kutta method to estimate $y(1)$ using just one step (i.e., use $h = 1$). Which estimate do you think is the more accurate? Why? *(16 points)*

Euler Method

Runge-Kutta Method