1. Find the general solution of $y'' + 2y' + 5y = e^{-2t}$ by first solving the corresponding undriven equation, then using the method of undetermined coefficients. (14 points)

2. Is it possible for $\{e^t, \sin(t)\}$ to form a basic set of solutions on the interval [0,4] for a differential equation of the form y'' + a(t)y' + b(t)y = 0 where a(t) and b(t) are both continuous on [0,4]? Explain. (10 points)

3. A rod of negligible mass is suspended from a high ceiling on a pivot. Attached to the bottom of the rod is a large mass acting as a pendulum bob. Your job is to estimate the length of the rod by timing the period of the swinging pendulum with a stopwatch. If a small displacement of the bob from equilibrium results in an 8 second period, estimate the length of the rod and explain how you arrive at your answer. *(12 points)*

4. Consider the equation $y'' + y = \sec t \ (-\frac{\pi}{2} < t < \frac{\pi}{2})$. Begin the variation of parameters method to find the general solution, but stop once you have computed $u'_1(t)$ and $u'_2(t)$ in the interest of saving time. (14 points)

5. Use the definition of the Laplace transform to find the Laplace transform of e^{3t} . Be sure to evaluate the improper integral carefully so your work can be followed. *(12 points)*

6. Find the Laplace transform of the solution of the IVP y'' + 2y' + 2y = sin(t), y(0) = 1, y'(0) = -1. (12 points)

7. The orbit diagram for a pendulum is given below. Describe the motion of the pendulum and the IVP that produced the orbit diagram (linear or non-linear, damped or undamped, estimate the initial conditions, tumbles over top of pivot or not – if so how many times, etc.). (*12 points*)



8. A solution y(t) of a driven mass-spring problem is given in the graph below. What can you tell about the mass-spring system from this solution (initial conditions, overdamped, characteristics of driving force, etc.)? Sketch the orbit diagram for this motion. (14 points)

