| MA 238-02 |  |  |  |
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| §3.1-3.7,4.1 <br> §4.2,4.4,6.1,6.2 | TeSt \#2 | score |  |

1. Find the general solution of $y^{\prime \prime}+2 y^{\prime}+5 y=e^{-2 t}$ by first solving the corresponding undriven equation, then using the method of undetermined coefficients. (14 points)
2. Is it possible for $\left\{e^{t}, \sin (t)\right\}$ to form a basic set of solutions on the interval [0,4] for a differential equation of the form $y^{\prime \prime}+a(t) y^{\prime}+b(t) y=0$ where $a(t)$ and $b(t)$ are both continuous on $[0,4]$ ? Explain. (10 points)
3. A rod of negligible mass is suspended from a high ceiling on a pivot. Attached to the bottom of the rod is a large mass acting as a pendulum bob. Your job is to estimate the length of the rod by timing the period of the swinging pendulum with a stopwatch. If a small displacement of the bob from equilibrium results in an 8 second period, estimate the length of the rod and explain how you arrive at your answer. (12 points)
4. Consider the equation $y^{\prime \prime}+y=\sec t\left(-\frac{\pi}{2}<t<\frac{\pi}{2}\right)$. Begin the variation of parameters method to find the general solution, but stop once you have computed $u_{1}^{\prime}(t)$ and $u_{2}^{\prime}(t)$ in the interest of saving time. (14 points)
5. Use the definition of the Laplace transform to find the Laplace transform of $e^{3 t}$. Be sure to evaluate the improper integral carefully so your work can be followed. (12 points)
6. Find the Laplace transform of the solution of the IVP $y^{\prime \prime}+2 y^{\prime}+2 y=\sin (t), y(0)=1$, $y^{\prime}(0)=-1$. (12 points)
7. The orbit diagram for a pendulum is given below. Describe the motion of the pendulum and the IVP that produced the orbit diagram (linear or non-linear, damped or undamped, estimate the initial conditions, tumbles over top of pivot or not - if so how many times, etc.). (12 points)

8. A solution $y(t)$ of a driven mass-spring problem is given in the graph below. What can you tell about the mass-spring system from this solution (initial conditions, overdamped, characteristics of driving force, etc.)? Sketch the orbit diagram for this motion. (14 points)


