Distribution Problems[1]

Domain (k)	**Range** (n)	**may receive 0**	**each receives ≥ 1**
Distinct	Distinct	*functions* n^k	*surjections* $n!S(k,n)$
Distinct	Distinct (each receives ≤ 1)	*injections* $P(n,k)$	*bijections* $n!$ if $n = k$ 0 if $n \neq k$
Distinct	Distinct (order received matters)	*ordered distributions* $P(n+k-1,k)$	$k!\binom{k-1}{n-1} = k!\binom{k-1}{k-n}$
Distinct	Identical	$\sum_{i=0}^{n} S(k,i)$	$S(k,n)$
Distinct	Identical (each receives ≤ 1)	1 if $k \leq n$ 0 if $k > n$	1 if $k = n$ 0 if $k \neq n$
Distinct	Identical (order received matters)	$\sum_{i=0}^{n} L(k,i)$	*broken permutations* $L(k,n) = \binom{k}{n}P(k-1,n-1)$
Identical	Distinct	$\binom{k+n-1}{k}$	$\binom{k-1}{k-n}$ if $k \geq n$ 0 if $k < n$
Identical	Distinct (each receives ≤ 1)	*subsets* $\binom{n}{k}$ if $k \leq n$ 0 if $k > n$	1 if $n = k$ 0 if $n \neq k$
Identical	Identical	$\sum_{i=0}^{n} Part(k,i)$ if $n < k$ $Part(k)$ if $k \leq n$	$Part(n,k)$
Identical	Identical (each receives ≤ 1)	1 if $k \leq n$ 0 if $k > n$	1 if $k = n$ 0 if $k \neq n$

1. $P(n,k)$ denotes the number of k-permutations of n objects.
2. $S(n,k)$ denotes the *Stirling numbers of the Second Kind*, i.e., the number of set partitions of an m-element set into n classes.
3. $L(k,n)$ denotes the *Lah numbers* or number of *broken permutations*, i.e., the number of ways to break up k distinct objects into n unordered classes of non-empty permutations.
4. $Part(k,n)$ denotes the number of integer partitions of an integer n into k parts.
5. $Part(n)$ denotes the total number of integer partitions of n.

[1] *Introductory Combinatorics*, Third Edition, by Kenneth P. Bogart