Matching Numbers in a Lottery

The number of ways of selecting 6 numbers from the numbers 1 through 49 (without replacement) is

$$C(49,6) = \frac{49!}{6!(49-6)!} = 13,983,816$$

Now supose that 6 numbers have been selected as above by Player A. The number of ways that Player B may choose 6 numbers between 1 and 49 (without repetition) so that exactly one of the chosen numbers matches one of the six selected by Player A is

$$C(6,1) \cdot C(43,5) = 6 \cdot 962,598 = 5,775,588$$

The first factor is the number of ways Player B can select one number from the 6 selected by Player A. The second factor is the number of ways the 5 remaining numbers can be selected by Player B so that none matches the remaining 5 numbers chosen by Player A.

Similarly, the number of ways exactly two of the six numbers chosen by Player B match those selected by Player A is

$$C(6,2) \cdot C(43,4) = 15 \cdot 123,410 = 1,851,150$$

The number of ways there are exactly three matches is

$$C(6,3) \cdot C(43,3) = 20 \cdot 12,341 = 246,820$$

The number of ways there are exactly four matches is

$$C(6,4) \cdot C(43,2) = 15 \cdot 903 = 13,545$$

The number of ways there are exactly five matches is

$$C(6,5) \cdot C(43,1) = 6 \cdot 43 = 258$$

And, of course, the number of ways all six match is 1.

To compute probabilities of of the above events, just take quotients of the number of occurrences of the events with $C(49,6)$. For example, the probability of choosing exactly one correct number is

$$\frac{5,775,588}{13,983,816} \approx 0.413019$$

EXERCISE: Calculate the probabilities of choosing two correct numbers, three, etc., in this lottery.