

MA 367-01 §5.5 – 6.1	<b>Quiz #3</b>	score	Name: _____ 10 February 2003
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INSTRUCTIONS: Turn in solutions to the following problems by Friday (14 February 2003) in class. As usual, fully explain your solutions and calculate the numerical values (when appropriate).

1. Show that

$$\left[ \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} \right]^2 = \sum_{k=0}^{2n} \binom{2n}{k}.$$

2. Evaluate

$$\sum_{k=1}^n (-1)^k k \binom{n}{k}.$$

3. Enumerate all the permutations of the letters  $a, b, c, d$

(a) in lexicographic order.

(b) Since that was so much fun, do it again in a minimum change order this time, i.e., so that each permutation is obtained from its predecessor by interchanging a single pair of letters. Better yet, make the last permutation in the list differ from the first by a single interchange, also.

4. Construct a generating function for  $a_r$ , the number of distributions of  $r$  identical objects into:

(a) five different boxes with at most 4 objects in each box;

(b) four different boxes with between 3 and 8 objects in each box;

(c) seven different boxes with at least one object in each box.

You may leave your answers in factored form on this problem.

5. Construct a generating function that could be used to determine how many ways there are to distribute 10 identical balls into 4 different boxes so that the first box has between 2 and 6 balls, the second box has an odd number of balls, and the other two boxes have no more than 6 balls each. The use a computer algebra system to determine the numerical answer.