| MA 367-01 <br> §7.1, 7.3, 7.4 | QuiZ \#4 |  | Name: $\frac{}{}$ |
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Instructions: Turn in solutions to the following problems by Wednesday (12 March 2003) in class. As usual, fully explain your solutions and calculate the numerical values (when appropriate). You should work exactly two of the problems numbered 2, 3 , and 4 (your choice).

1. Find a recurrence relation with initial conditions for the number of $n$-digit ternary sequences
(a) that don't have any consecutive 0 's;
(b) that have an even number of 0 's.
2. The integers $1,2, \ldots, n$ are pushed onto a LIFO stack in order ( 1 first, etc.). Each integer is ultimately popped off the stack. Once a particular integer is popped, it cannot be pushed back on the stack. The pushes and pops can be interlaced. How many different permutations of the integers can be constructed by this procedure? Do this first for $n=2$, $n=3$, and maybe $n=4$. Then show how the case for arbitrary $n$ can be counted. Explain fully.
3. Given a convex $n$-sided polygon, in how many ways can it be traingulated using diagonals that do not intersect except at a vertex. Explain fully.
4. Determine a general formula for the number of rooted binary trees with $n$ vertices. You might begin by looking up the definition of rooted binary tree, and then counting specific cases like $n=2,3$ and maybe $n=4$. Explain fully.
5. Find a closed form solution to the recurrence relation $a_{n}=a_{n-1}+6 a_{n-2}$ with initial conditions $a_{0}=1, a_{1}=2$. Show the complete solution method.
6. Find a closed form solution to the recurrence relation $a_{n}=2 a_{n-1}+n$ with initial condition $a_{0}=2$. Show the complete solution method.
