MA 507-51 §3.8–7.3	**Test #2**	*score*	**Name:** ______________ **13 November 2000**

INSTRUCTIONS: Work these problems on a separate sheet. Submit the solutions in order by Wednesday, November 22, 2000. The department office will close at noon that day.

1. Consider the forced undamped harmonic oscillator given by

$$x'' + 4x = \cos(\Omega t).$$

For what value of Ω does resonance occur? Graph a solution that demonstrates resonance (make up some initial conditions) and also graph a solution that demonstrates the phenomenon of beats using a value of Ω slightly different from the resonant value. *(15 points)*

2. Consider the IVP $x'' + 0.5x' + 9x = f(t)$ with initial conditions $x(0) = 1$ and $x'(0) = 0$. We want to apply an impulse of force (using constant multiples of Dirac delta functions) to the system every 10 seconds beginning at $t = 10$ so that x returns approximately to its starting position of 1. Determine the coefficient that should be used on the Dirac functions to accomplish this. *(15 points)*

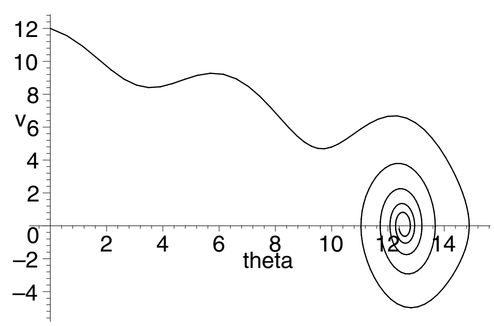
3. Use the Euler method and the Runge-Kutta method (fourth order) to approximate the value of the solution at $x = 1$ for the following IVP.

$$y' + xy^2 = 1, \quad y(0) = .5$$

In both cases, use a step-size of 0.5. Do the calculations by hand and show your work. Then use Maple to compute the exact solution (it will use various special functions) and graph the solution on the [0,1] interval. Plot the points you calculated to see how close the approximation methods were and compare their values with the values of the exact solution. (You can plot the points by hand on the Maple graph of the solution if you have trouble getting Maple to do it.) *(15 points)*

4. A rod of negligible mass is suspended from a high ceiling on a pivot. Your job is to estimate the length of the rod, but the only tools you have available are a 1kg mass and a stopwatch. So you attach the mass to the end of the rod and find that, with a small displacement, the resulting pendulum completes 8 swings in 20 seconds. Estimate the length of the rod and explain how you arrive at your answer. *(15 points)*

5. For the given pendulum orbit diagram, describe the motion of the pendulum and the IVP that produced the orbit diagram (linear or non-linear, damped or undamped, estimate the initial conditions, tumbles over top of pivot or not – if so how many times, etc.). *(15 points)*



6. Suppose we have a frictionless pendulum of length 1 meter with a mass attached to the end. If the mass is set in motion, what is the shortest period the pendulum can have? Use Maple to approximate the period of the pendulum if the initial angle is 90° and also 135° (released from rest in both cases). Assume the pendulum is released from rest in both cases.

Use Maple to draw a phase portrait (on one set of axes) for the pendulum using initial conditions where $\theta = 0$ and $\theta' = 1, 2, 3, \ldots 10$. Explain what the phase portrait shows. Then use Maple to draw similar a phase portrait for the corresponding pendulum with a damping coefficient of 0.5. Again explain what the phase portrait shows. *(25 points)*