MA 125-06 §2.1–5.3	**Final Exam**	*score*	**Name:** ____________ **11 December 2000**

1. Use the limit definition of derivative to calculate the derivative of the function $f(x) = \sqrt{x-1}$. *(7 points)*

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2. A particle moving along a straight number line is observed at the follwing locations. Estimate the velocity of the particle at time $t = 3$ seconds. *(7 points)*

time (seconds)	1	2	3	4	5
location (meters)	24	19	15	13	12

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3. Determine where the given function is continuous. Explain fully. *(7 points)*

$$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 3x & \text{for } 0 \le x < 2 \\ \sin(\pi x) + 5 & \text{for } x \ge 2 \end{cases}$$

4. Evaluate the following limits if they exist. Give reasons where appropriate. *(5 points each)*

   (a) $\displaystyle\lim_{x\to 2^-} \frac{\tan x}{(x-2)^3}$

   (b) $\displaystyle\lim_{x\to 0} \frac{[\![x]\!]}{x}$

5. Let C be the curve in the xy-plane described by the parametric equations $x(t) = \cos t$ and $y(t) = 2\sin t$. Find a formula, in terms of t, for the slope of the tangent line. Then determine all points on the curve that have a tangent line with slope -2. *(7 points)*

6. Let $h_1(x) = f(g(x))$, $h_2(x) = f(x)\cdot g(x)$, and $h_3(x) = \frac{f(x)}{g(x)}$. Use the information in the table to find the value of $h_1'(3)$, $h_2'(3)$, and $h_3'(3)$. *(7 points)*

x	1	2	3	4	5
$f(x)$	5	2	1	3	1
$f'(x)$	2	1	2	2	3
$g(x)$	3	2	4	3	1
$g'(x)$	3	2	3	2	3

7. Use implicit differentiation to find a formula for $\frac{dy}{dx}$ if $(xy)^2+x^2-y^3 = 1$. Then determine the slope of the tangent line to the curve when $x = 0$. *(7 points)*

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8. Two straight highways meet at a right-angle intersection. One car approaches the intersection from the north at a constant rate of 60 km/hr while another moves away from the intersection traveling east at a constant rate of 80 km/hr. At a particular moment, both cars are 2 km from the intersection. At what rate is the distance between the cars changing at that moment? *(8 points)*

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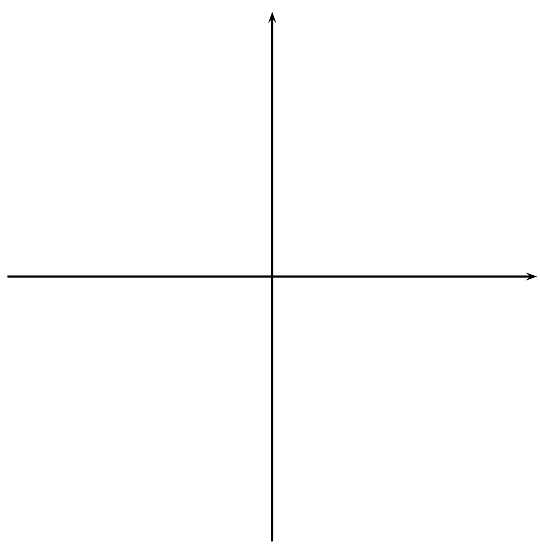
9. Let $f(x) = 1 - 3x + 5x^2 - x^3$. Find the intervals of monotonicity, the intervals of concavity, the relative extrema and the invlection points. Sketch the graph of $f(x)$ and label the important points. *(8 points)*



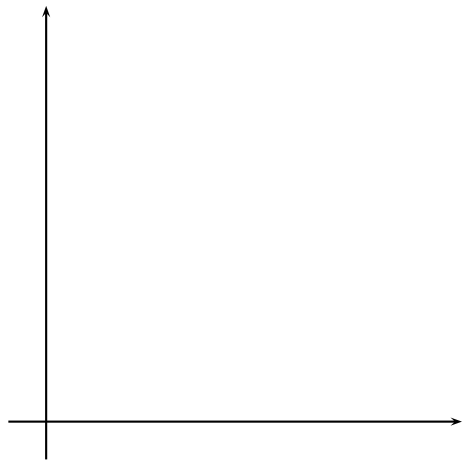
10. Evaluate the following limits. Show your work and explain as needed.

    (a) $\displaystyle\lim_{x\to 0} \frac{e^{2x}-1}{\sin x}$

    (b) $\displaystyle\lim_{x\to \infty} \frac{\ln \ln x}{\sqrt{x}}$

11. A right triangle has vertices (0,0), (3,0), and (0,5). Find the area of the largest rectangle that can be inscribed in the triangle if two edges of the rectangle are along the legs of the triangle. *(8 points)*

12. Let $f(x) = \cos x$. Draw a sketch of $f(x)$ together with the rectangles used in the Riemann sum from $x = 0$ to $x = \frac{\pi}{2}$ based on left endpoints with 4 subintervals. Then use your calculator to compute the values of the left sums using 4, 10, and 50 rectangles. *(7 points)*



13. A particle has a velocity function of $v(t) = \dfrac{1}{1+t^2}$. If the particle begins at $x = 10$ when $t = 0$, where will it be located at $t = 1$ and $t = 10$? (Approximate your answers to two decimal places.) *(7 points)*