Project 1 Solution

MA 125-06

Fall 2000

INSTRUCTIONS: Each team will submit one report. All members of the team get the same grade. Each team member must sign the report. The report should include a labeled drawing and full verbal explanations. Write up the solution as you would like it to appear in a well-written text. The report is due on Friday, November 3, 2000.

PROBLEM: A large clock located in a clock tower has an hour hand that is 1.5 meters long and a minute hand that is 2 meters long. At what rate is the distance between the tips of the hands changing at 2:00 o'clock?

SOLUTION: Let d be the distance between the tips of the two hands and θ the angle between them as shown in the diagram. Then from the law of cosines, we have

$$d^2 = 2^2 + 1.5^2 - 2(2)(1.5)\cos\theta \tag{1}$$

Differentiating both sides of the equation with respect to time t, we get

$$2dd' = 0 + 0 + 6(\sin\theta)\theta'$$

Solving for d' we have

$$d' = \frac{3(\sin\theta)\theta'}{d} \tag{2}$$

At the given moment (2:00 o'clock), $\theta = \dfrac{\pi}{3}$ radians/sec, $\theta' = (\frac{2\pi}{12} - 2\pi) = \frac{11\pi}{6}$ radians/hour (taking the difference of the angular speed of the two hands in radians per hour), and $d = \sqrt{2^2 + 1.5^2 - 2(2)(1.5)\frac{1}{2}} = \sqrt{3.25} = \dfrac{\sqrt{13}}{2} \approx 1.8$ meters from (1). Substituting these values into equation (2), we get $d' = \dfrac{3\sin(\pi/3)(-11\pi/6)}{\sqrt{13}/2} = \dfrac{-5\pi\sqrt{39}}{26} \approx -8.3$ meters/hour. So the distance between the tips of the hands is decreasing at a rate of 8.3 m/hour (or 0.138 meters/minute if you prefer those units).