Quiz #3 (§14.3 – §14.4, §14.7) **Name:** ____________________

MA 234-51 **28 January 1993**

---

1. Find the length of the curve given by $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + \frac{2}{3}t^3\,\mathbf{k}$ from the point $(0,0,0)$ to the point $(3,9,18)$. *(7 points)*

   Note that the indicated part of the curve is swept out from $t=0$ to $t=3$. Thus,

$$\begin{aligned} L &= \int_0^3 \|\mathbf{r}'(t)\|\,dt = \int_0^3 \sqrt{1^2+(2t)^2+(2t^2)^2}\,dt \\ &= \int_0^3 \sqrt{1+4t^2+4t^4}\,dt = \int_0^3 1+2t^2\,dt = \left(t+\frac{2}{3}t^3\right)\Big|_0^3 \\ &= 3+18 = 21 \end{aligned}$$

---

2. Let $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + 4t\,\mathbf{k}$.

   (a) Find the unit tangent and unit normal vectors at the point $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 3\pi\right)$. *(7 points)*

   Note that $t = \frac{3\pi}{4}$ at the given point.
   $\mathbf{r}'(t) = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j} + 4\,\mathbf{k}$, so
   $\mathbf{T}(t) = \frac{1}{\sqrt{17}}(-\sin t\,\mathbf{i} + \cos t\,\mathbf{j} + 4\,\mathbf{k})$.
   $\mathbf{T}'(t) = \frac{1}{\sqrt{17}}(-\cos t\,\mathbf{i} + -\sin t\,\mathbf{j} + 0\,\mathbf{k})$, so
   $\mathbf{N}(t) = -\cos t\,\mathbf{i} - \sin t\,\mathbf{j}$. Thus, at the given point, we have
   $\mathbf{T}(\frac{3\pi}{4}) = \frac{1}{\sqrt{17}}\left(-\frac{\sqrt{2}}{2}\,\mathbf{i} - \frac{\sqrt{2}}{2}\,\mathbf{j} + 4\,\mathbf{k}\right)$, and
   $\mathbf{N}(\frac{3\pi}{4}) = \frac{\sqrt{2}}{2}\,i - \frac{\sqrt{2}}{2}\,\mathbf{j}$.

   (b) Find the value of the curvature at every point on the curve. *(6 points)*

$$\begin{aligned} k &= \frac{\|\mathbf{r}'\times\mathbf{r}''\|}{\|\mathbf{r}'\|^3} = \frac{\|(-\sin t, \cos t, 4)\times(-\cos t, -\sin t, 0)\|}{\|(-\sin t,\cos t,4)\|^3} \\ &= \frac{\|(4\sin t, -4\cos t, \sin^2 t+\cos^2 t)\|}{(\sin^2 t+\cos^2 t+16)^{3/2}} = \frac{\sqrt{16\sin^2 t+16\cos^2 t+1}}{(17)^{3/2}} \\ &= \frac{1}{17} \end{aligned}$$