MA 367-01 §5.1–6.3	**Test #1**	*score*	**Name:** ______________ **24 February 2003**

INSTRUCTIONS: You do not need to compute numerical values for your answers, but you must express them using using arithmetic operations and the factorial operator. Explain how you obtain your answers unless otherwise directed.

1. *(25 points)* SHORT ANSWER: You just need to write the answer on this one – no explanation is necessary.
   (a) If $|A| = n$, the total number of subsets of A is ______________.
   (b) If $|A| = n$, the number of subsets of A of size k is ______________.
   (c) The number of distinguishable ways can 3 A's, 4 B's, and 6 C's be all arranged in a row is ______________.
   (d) The number of ways of selecting k objects without repetition from n distinguishable objects ($k \le n$) and arranging them in a row is ______________.
   (e) The infinite series $1 + x + x^2 + x^3 + \cdots$ can be formally written in closed form as ______________.

2. *(15 points)* How many ways can a group of 5 girls and 4 boys be lined up in a row
   (a) (with no restriction)?
   (b) if the boys all have to be adjacent and the girls have to be adjacent?
   (c) if no two girls can be adjacent?

3. *(14 points)* A coin is tossed 12 times.
   (a) How many ways can this result in 9 heads and 3 tails?
   (b) Of those, how many contain at least 5 heads in a row?

4. *(11 points)* In how many ways can 10 identical balls be distributed into 4 distinguishable boxes so that the first box has an odd number of balls, the second box has an even number of balls, and the last two boxes have between 2 and 5 balls (inclusive)? Don't actually compute this number, but describe how it could be computed using generating functions.

5. *(15 points)* How many ways can 12 jobs be assigned to 3 different people if the jobs are indistinguishable for counting purposes
   (a) ?
   (b) if each person gets at least one one job?
   (c) if each person gets at least three jobs?

6. *(10 points)* Use the binomial theorem to show that the number of subsets of a set A having an odd number of elements is the same as the number of subsets of A having an even number of elements.

7. *(10 points)* Let $p(m, n)$ denote the number of integer partitions of m into exactly n parts. Complete the following table of values for $p(m, n)$ using the property that a value in a

row can be expressed as a sum of values in a particular previous row. Explain in detail how $p(8,3)$ is computed as a specific example.

m / n	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
1	1	0	0	0	0	0	0	0
2	1	1	0	0	0	0	0	0
3	1		1	0	0	0	0	0
4	1			1	0	0	0	0
5	1				1	0	0	0
6	1					1	0	0
7	1						1	0
8	1							1