

MA 367-01 §7.1-8.3	Test #2	score	Name: _____ 23 April 2003
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INSTRUCTIONS: Put all answers/solutions on a separate sheet. Explain how you obtain your answers unless otherwise directed.

- (36 points) SHORT ANSWER: Answer each of the following and explain what you did.
 - In how many ways can 5 pairs of parentheses be arranged in a row that are algebraically correct?
 - Write the rook polynomial for the board

X	X
X	X

 (the X denotes shading).
 - Compute a Stirling Number of the Second Kind $S(6, 3)$, and give a combinatorial explanation of what this number counts.
 - Compute the derangement number, D_{10} , the number of derangements of 10 objects.
 - Find the number of onto functions $f : \mathbb{N}_5 \rightarrow \mathbb{N}_3$.
 - If the rook polynomial for the shaded (forbidden) positions in a 5×5 chessboard is $1 + 13x + 56x^2 + 96x^3 + 60x^4 + 12x^5$, in how many ways can 5 mutually non-attacking rooks be placed on the unshaded squares?
- (16 points) Let a_n denote the number of n -digit ternary (digits are 0, 1, or 2) sequences that do not have two consecutive zeros. Determine (and fully explain) a recurrence relation with initial conditions for this sequence. Do not solve this recurrence relation.
- (16 points) Find the general solution for the recurrence relation $a_n = 3a_{n-2} + 2a_{n-1}$. Then find the solution that satisfies the initial conditions $a_0 = 3$ and $a_1 = 1$.
- (16 points) Use inclusion-exclusion to find the number of ways to distribute 20 identical pieces of candy among four combinatorics students if no student can get no more than 5 pieces of candy. Show the details of your inclusion-exclusion counting method.
- (16 points) Find the rook polynomial for the following forbidden position problem. You may leave the polynomial in factored form, and you need not go any farther with the problem than finding the rook polynomial.

We want to find the number of ways 5 people (A, B, C, D and E) can be assigned 5 tasks (1, 2, 3, 4 and 5) to do if person A cannot do tasks 1 and 2, person B cannot do tasks 2 and 4, person C cannot do tasks 1 and 2, and person D cannot do tasks 3 and 4, and person E cannot do tasks 4 and 5.